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Homogeneously Weighted Moving Average Control Chart for Rayleigh Distribution

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Abstract

In this paper, we have proposed Homogeneously Weighted Moving Average (HWMA) control chart for Rayleigh distribution. The Average Run Length (ARL_1) is used to evaluate the performance of the proposed HWMA control charts. The ARL_1 performance of HWMA control chart is compared to the Exponentially weighted moving average (EWMA) control charts with respect to the different shift size (i.e. 10%, 15%, 20%, 30%, 40% increase and decrease in shift). The results are calculated using sample size n=5. It is observed that with the increase in shift proposed HWMA chart shows more efficient results i.e. ARL_1 values decrease with the increase in shifts. It is found that the proposed HWMA chart for Rayleigh distribution outperforms the existing EWMA control chart.

Keywords: HWMA chart, Average Run Length, Rayleigh distribution, EWMA, Shifts

1. Introduction

Control charting is the most effective way for managing a process in SPC. Control charts were first proposed by Shewhart (1931). Shewhart's control charts are based on the premise that the data will follow a Normal distribution. The 99.73% of observations sampled from a Normal distribution are included within 3 standard deviations of the mean, i.e. between the limits $\mu \pm 3\sigma$. If the observation plots within these boundaries, the process is said to be under control; if the observation plots beyond these limits, the process is said to be out of control. The 3-sigma limits are a simple and effective technique for determining if a process is under control. We may calculate the control limits of the control chart using the 99.73 quantile point for the distribution under investigation when the distribution is non-normal and follows any other skewed distribution. (Montgomery, 2009).

Aczel (1989) stated that a process capacity is its normal behavior once disruptions are removed. Montgomery (2009) states that the processes operate in the presence of assignable/special causes might be categorised as out of control.

Abbas at al. (2012) stated that Control charts are the most frequently used method for detecting the existence of Special Causes of Process Changes. Memory and memory-less control charts are the two forms of control charts. Unlike memoryless control charts, such as Shewhart type control charts, memory control charts, such as cumulative sum and exponentially weighted moving average control charts, are designed in such a way that past data is not lost.

Ahmed (2014) describes that adding auxiliary information to control charts improves their detection capabilities, allowing for more efficient monitoring of process parameter(s). Their research focuses on Shewhart type variability control charts for non-cascading systems based on auxiliary features, with auxiliary values assumed to be stable.

Azam et al. (2015) presented a cyclical sampling-based hybrid exponentially weighted moving average (HEWMA) control chart. They suggest that the control chart is for a normally distributed quality criterion. In light of the goal in-control ARL, control chart coefficients are determined.

Aslam et al. (2016) presented the entire design of a COM-Poisson control chart using a hybrid exponentially weighted moving average (HEWMA).

Abbasi and Riaz (2016), described that statistical process is controlled by employing dual auxiliary information, both for ranking and estimation. The control charts are suggested in three fundamental ranked set sampling schemes: ranked-set sampling (RSS), median RSS, and extreme RSS.

Abbas (2018) suggested a homogeneously weighted moving average control chart that may be used in the substrate fabrication process. The suggested control charting statistic gives the current sample a specific weight while the remaining weight is evenly divided across the prior samples.

Adegoke at al. (2019) proposed Efficient Homogeneously Weighted Moving Average Chart for Monitoring Process Mean Using an Auxiliary Variable. For cases in which the process variable is monitored with a correlated auxiliary variable, they suggested an efficient control chart for monitoring minor variations in the process mean.

Abdul Haq and Lubna Bibi (2020) proposed a new dual CUSUM mean chart. They showed dual Crosier CUSUM (DCCUSUM) charts with and without quick initial response characteristics to monitor rare changes in the mean of a normally distributed process effectively.

Butt and Raza (2017) explain the performance of the Exponentially Weighted Moving Average (EWMA) control chart in the presence of Type I censored data.

Chen et al. (2004) developed a new control chart based on the exponentially weighted moving average (EWMA) technique. The control region was defined by the statistic for the chart as the area below a straight line, which simplifies the charting method.

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Grant Leavenworth (1980) describes "Statistical Process Control (SPC) is a valuable and significant technique used regularly in the manufacturing industry to monitor the whole process,". SPC may be used in a variety of engineering situations. The process will become more consistent and dependable as a result of the extensive use of SPC analysis.

Gupta and Gupta (2009) describes statistical quality control is one of the most beneficial and cost-effective uses of sampling theory in the industrial area.

Haq (2012, 2013, 2016) proposed a new hybrid EWMA (HEWMA) control chart which is presented by combining two EWMA control charts. Control charts such as the cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) are widely used to monitor the process mean.

Haq and Khoo (2017) proposed that "Statistical quality control charts have been widely regarded as a potentially strong process monitoring tool due to their outstanding speed in detecting variations in the underlying process parameter(s), Auxiliary-information-based (AIB) control charts have outperformed non-AIB control charts in recent tests in terms of run-length performance."

Javaid et al. (2018) developed a control chart utilizing an auxiliary variable for combined monitoring of the process mean and variance.

Mandel (1969) proposed the linear regression based control chart theories which provided an efficient approach for real life applications.

Keller (2015) and Mandel (1969) described Quality control as "Statistical Process Control which it refers to one of a number of statistical approaches used to build and maintain a firm's capacity to provide high-quality goods and services."

Kotz and Johnson (1988) defined the reasons of variations. It might be the result of random (chance) or assignable factors. Any given scheme of production and check has certain stable systems of chance.

Munir and Haq (2017) suggested "a new cumulative sum (CUSUM) control chart for monitoring the variability of a normally distributed process utilising the ordered ranked set sampling (RSS) and ordered double RSS methods."

Abid at al. (2020) describes a mixed cumulative sum homogeneously weighted moving average (HWMA) control chart for monitoring the process mean. They demonstrated real-world applications of existing and planned charts based on data from Major League Baseball's home run per game (HRPG) datasets (MLB). Abid et al. (2020) also suggested a new double HWMA (DHWMA) chart for monitoring the changes in the process mean.

Maonatlala Thanwane et al. (2020) suggested the Homogeneously Weighted Moving Average Chart's Parameter Estimation Effect to Monitor the Mean of Autocorrelated Observations With Measurement Errors

Nurudeen A. Adegoke (2021) describes a Directionally sensitive homogeneously weighted moving average control charts to monitor changes in the mean level for one-sided control charts,.

Raza and Butt (2018) presented two new control charts. The Shewhart-type control chart and the EWMA-type control chart are two different types of control charts. The control charts suggested are based on Noor-ul-Amin and Hanif's (2012) exponential type mean estimator.

Ali and Haq (2018) developed a new GWMA-CUSUM control chart to monitoring process dispersion. They suggested a CUSUM chart for monitoring process dispersion using the generally weighted moving average (GWMA) statistic, named the GWMA-CUSUM chart, to improve the CS-EWMA chart's detection capabilities.

Raji et al. (2018) developed a double exponentially weighted moving average chart. Singh and Vishwakarma (2007) proposed and analyzed exponential ratio and product estimators for estimating finite population mean in double sampling using auxiliary information.

Shabbir and Awan (2015) presented the Shewhart control charts are not highly sensitive to small and moderate size process shifts. Cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts are used as an alternative to Shewhart control charts in Phase II to monitor tiny or reasonable process changes.

Saghir et al. (2018) provided an exponentially weighted moving average (EWMA) control chart with an auxiliary variable and recurrent sampling for efficient detection of minor to moderate variations in position. The outer and inner control limits for the intended in-control average run duration are used to establish the control chart coefficients of the proposed EWMA chart.

Woodall et al. (2004) assumed that the quality of a process or product can be adequately represented by the distribution of a univariate quality characteristic or the general multivariate distribution of a vector consisting of several correlated quality characteristics in statistical process control (SPC) applications.

Vasileios Alivizakos et al. (2021) suggested the extended homogeneously weighted moving average control chart. They imitating exactly the double EWMA (DEWMA) approach to expand the HWMA chart.

1.1. Exponentially Weighted Moving Average (EWMA) Control chart

The EWMA chart is a form of control chart used in statistical quality control to monitor variables or attributes-type data over the whole history of output from the monitored business or industrial process. The EWMA chart monitors the exponentially weighted moving average of all preceding sample means, whereas Shewhart-type control charts address reasonable subsets of samples separately. The EWMA algorithm weights data in a geometrically decreasing sequence, with the most recent samples receiving the highest weighting and the most distant samples contributing very little. Although the normal distribution is the basis of the EWMA chart, the chart is also relatively robust in the face of non-normally distributed quality characteristics (Montgomery, 2009).

Instead of plotting rational subgroup averages directly, the EWMA chart computes successive observations z_i by computing the rational subgroup average, $\bar{x}_i$, and then combining that new subgroup average with the running average of all preceding observations, zi - 1, using the specially–chosen weight, λ, as follows:

$$z_i = \lambda \bar{X}_i + (1-\lambda) z_{i-1} \qquad (1)$$

The control limits for this chart are defined as:

$$T \pm L \frac{S}{\sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]} \qquad (2)$$

where T and S are the estimates of the long-term process mean and standard deviation established during control-chart setup and n is the number of samples in the rational subgroup.

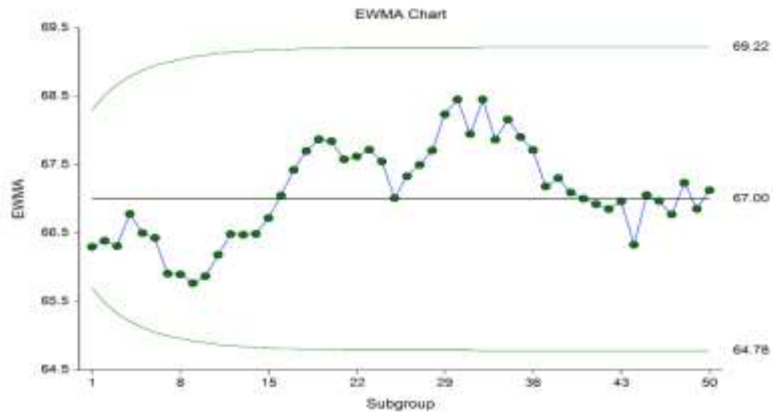

Figure 1: Traditional/simple EWMA chart

1.2. Homogeneously weighted moving average (HWMA) Control Chart

The homogeneously weighted moving average (HWMA) control chart is a new memory-type chart that allocates a specific weight to the current sample and the remaining weight is distributed equally to the previous samples.

$$H_i = \omega \overline{X_1} + (1-\omega)\,\bar{\bar{X}}_{i-1} \qquad (3)$$

The control limits for this chart are defined as:

$$\mu_0 - C\sqrt{\frac{\omega^2 \sigma_0^2}{n}} \pm \mu_0 + C\sqrt{\frac{\omega^2 \sigma_0^2}{n}} \qquad (4)$$

1.3. Average Run Length (ARL)

The Average Run Length is used to evaluate the performance of a control chart (ARL). “The average number of points drawn on a control chart before an out of control condition is signaled is known as the Average Run Length. One of the finest alternatives for detecting assignable cause will be the control chart with the lowest ARL1 value. (Montgomery, 2009).

All procedures have some variation, according on the real-life applications. However, the process can occasionally exhibit significant inconsistency, resulting in offensive or surprising outcomes. The SPC is used to reduce variation in order to achieve the best objective value. (Awan and Shabbir, 2014).

1.4. Objectives of the study

The objective of this paper are:

- To propose Homogeneous Weighted Moving Average (HWMA) control chart for monitoring Rayleigh life times.
- To develop the control charting parameters, consisting of an upper control limit (UCL), a lower control limit (LCL), a centerline (CL).
- To compare the proposed control charts with the existing EWMA control charts.

1.5. Research Questions

The study is required to answer the following question:

➢ Does the proposed HWMA control chart for Rayleigh distribution shows more efficient results than the existing EWMA Control chart?

1.6. Research Hypothesis

The following are the alternative hypothesis:

H_{1a}: The proposed HWMA control chart for Rayleigh distribution shows more efficient results than the existing EWMA Control chart.

2. Methodology

2.1. The Proposed HWMA Control Chart

In this paper, we have proposed homogeneously weighted moving average (HWMA) control chart for Rayleigh distribution. The HWMA is a new memory-type chart that allocates a specific weight to the current sample and the remaining weight is distributed equally to the previous samples.

Let $X_{i,j} \sim Rayleigh\ (\sigma)$ be the quality characteristic to be monitored, where i=1,2,3,……m and j=1,2,3,……,n. Initially, we consider the population parameters σ to be known. The plotting statistic for HWMA chart is defined as:

$$\boldsymbol{T_i = \emptyset\overline{X_1} + (1\text{-}\ \emptyset)\ \overline{\overline{X}}_{i-1}}$$

where Xi is the sample average for ith sample. $\emptyset$ is the smoothing constant (called the sensitivity parameter of the HWMA chart) selected between zero and one i.e.

$0 < \emptyset \leqslant 1$. $\overline{\overline{X}}i$-1 is the mean of means of previous (i−1) samples and is given as:

$$\boldsymbol{\overline{\overline{X}}_{i-1} = \frac{\sum_{k=1}^{i=1}\overline{X_k}}{i=1}}$$

The value of $X_0$is set equal to the target mean of X i.e. μ_0. Ti can be rewritten as:

$$\boldsymbol{T_i = \emptyset\overline{X_\iota}} + [\left(\tfrac{1-\emptyset}{i-1}\right)\boldsymbol{\overline{X}_{i-1}} + \left(\tfrac{1-\emptyset}{i-1}\right)\boldsymbol{\overline{X}_{i-2}} + \cdots + \left(\tfrac{1-\emptyset}{i-1}\right)\boldsymbol{\overline{X}_1}]$$

The rationale of the statistic given in is to give $\emptyset$ weight to the current sample and the rest of the (1− $\emptyset$) weight is homogeneously distributed to all the previous samples. The control limits for the

HWMA chart can now be defined as:

$$\boldsymbol{LCL_i = E\,(T_i) - C}\,[\boldsymbol{Var\,(T_i)}],\ \boldsymbol{CL = E\,(T_i)},\ \boldsymbol{UCL_i = E\,(T_i) + C}\,[\boldsymbol{Var\,(T_i)}]$$

The mean and variance for Hi (i.e. E (Ti) and V (Ti), respectively) for an in-control situation are derived respectively. Using the mean and variance of Ti, the control limits of HWMA chart from become:

$$LCL_i = \begin{bmatrix} \mu_0 - C\sqrt{\dfrac{\emptyset^2\sigma_0^2}{n}}, & if\ i = 1 \\ \mu_0 - C\sqrt{\dfrac{\emptyset^2\sigma_0^2}{n} + (1-\emptyset)^2\dfrac{\sigma_0^2}{n(i-1)}}, & if\ i > 1 \end{bmatrix}$$

$$CL = \mu_0$$

$$UCL_i = \begin{bmatrix} \mu_0 + C\sqrt{\dfrac{\emptyset^2\sigma_0^2}{n}}, & if\ i = 1 \\ \mu_0 + C\sqrt{\dfrac{\emptyset^2\sigma_0^2}{n} + (1-\emptyset)^2\dfrac{\sigma_0^2}{n(i-1)}}, & if\ i > 1 \end{bmatrix}$$

where C determines the width of the control limits and it is decided based on the desired ARL_0.

2.2. Construction of Distance Weighted Mean Control Chart

The construction of the distance weighted control chart are given below:

1. 100 samples of size 5 are generated by using random numbers from Rayleigh distribution with specified value of σ.
2. HWMA charting statistics is computed for each of the 100 samples obtained in (1).
3. By using 99.73% quantile point approach, control limits are computed using results obtained in (2).
4. Increasing or decreasing of shifts in the specified parameter are introduced in next 100 samples.
5. The charting statistics is plotted against the control limits.
6. When the charting statistics fall outside the control limits we record the number as Run length (RL).
7. Now by using Monte Carlo Simulation, this whole process is repeated 10000 times and the 10000 RLs are obtained.
8. Now we take the average of run lengths to obtain the ARL_1 values.

2.3. Monte Carlo (MC) Simulations

The Monte Carlo Simulation, also known as the Monte Carlo Method or a multiple probability simulation, is a mathematical technique, which is used to estimate the possible outcomes of an uncertain event. The Monte Carlo Method was invented by John von Neumann and Stanislaw Ulam during World War II to improve decision making under uncertain conditions ("Monte Carlo Simulation", 2021).

A Monte Carlo Simulation builds a model of possible results by leveraging a probability distribution, such as a uniform or normal distribution, for any variable that has inherent uncertainty. It, then, recalculates the results over and over, each time using a different set of random numbers between the minimum and maximum values. In a typical Monte Carlo experiment, this exercise can be repeated thousands of times to produce a large number of likely outcomes ("Monte Carlo Simulation", 2021).

In our study, the Monte Carlo (MC) Simulations method has been used for the result computation and replications. A number of scholars and researchers used the MC simulation technique in their analysis. The "MCSIM.xls" add-in is used for calculation/replication of results 10000 times.

3. Results and Discussion

3.1. Simulated data results

To study the features of the proposed HWMA control chart, we have simulated the data and applied the HWMA chart for Rayleigh distribution. We have used some results for discussion purpose. 4.1- 4.8 Tables give the ARLs of the proposed HWMA control charts and 4.9- 4.16 Tables give the ARLs of the proposed existing EWMA control chart. Table 4.17- 4.24 give the comparison of

ARLs values for EWMA and HWMA control chart. We have considered n = 5 and λ= 0.25, λ=0.15 and λ=0.35 for increase shift and λ=0.25 for decrease shift and ARL_0 = 200 and ARL_0 = 100.

Table 1: ARL_1 values for HWMA Control Chart for n=5, ARL_0=200 and λ=0.25

Incontrol ARL0=200					
ARL1					
Shifts	10% increase	15% increase	20%increase	30% increase	40% increase
σ					
0.5	46.1	43.14	40.7	37.89	30.1
1	45.52	44.31	40.04	31.22	31.88
2	48.01	45.75	40.38	32.27	29.16
3	45.1	49.73	43.22	35	29.28
5	44.56	44.34	41.79	32.06	30.8
8	47.05	45.2	42.25	31.72	28.9
10	49.65	46.59	43.76	33.64	29.26

The table 1 describes the ARL_1 performance of proposed HWMA control chart by taking Rayleigh distribution using n=5, and ARL_0=200. We observed that the efficiency of the HWMA control chart increases (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively.

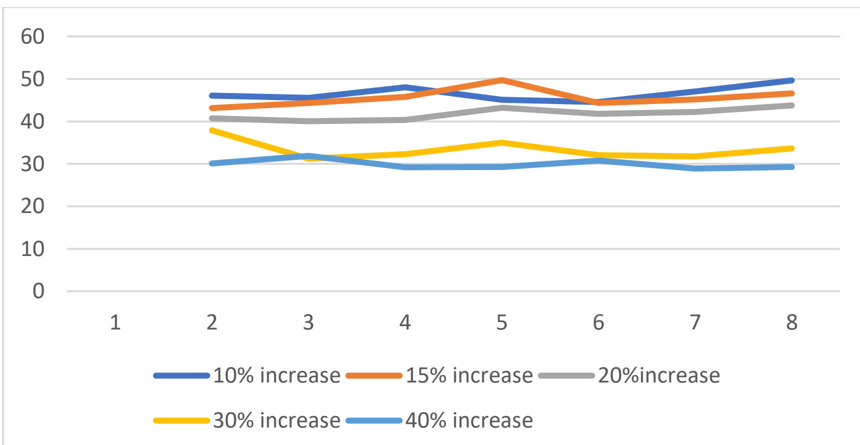

Figure 1: ARL_1 values for HWMA Control Chart for n=5, ARL_0=200 and λ=0.25

Figure 1 graphically shows the ARL_1 performance of HWMA control charts using n=5, ARL_0=200. We observed that the efficiency of the HWMA control chart increase (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively. The ARL_1values decrease for a fixed value of ARL_0 and λ's.

Table 2: ARL_1 values for HWMA Control Chart for n=5, ARL_0=100 and λ=0.25

Incontrol ARL0=100					
ARL1					
Shifts	10% increase	15% increase	20% increase	30% increase	40% increase
σ					
0.5	32.42	26.67	28.31	22.93	14.2
1	34.35	25.73	30.57	20.99	16.28
2	32.19	24.5	24.24	21.39	14.82
3	33.79	27.7	28.8	23.3	19.24
5	32.44	24.26	23.3	19.2	21.69
8	29.1	25.5	23.42	24.39	17.81
10	27.52	25.5	22.1	23.03	16.47

The table 2 describes the ARL_1 performance of proposed HWMA control chart by taking Rayleigh distribution using n=5, and ARL_0=100. We observed that the efficiency of the HWMA control chart increase (i.e. ARL_1 values decrease) with the increase in Shift from 10% to 40% respectively.

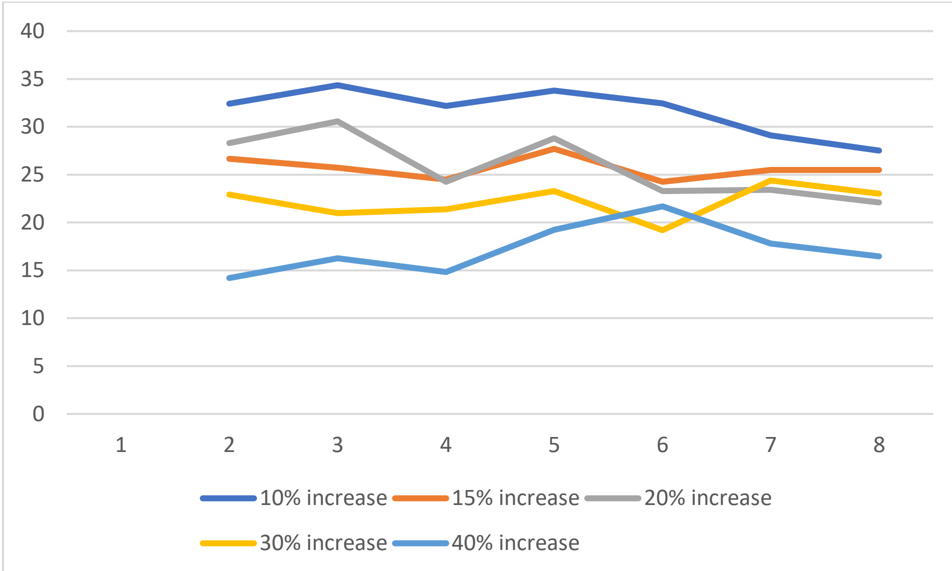

Figure 2: ARL_1 values for HWMA Control Chart for n=5, ARL_o=100 and λ=0.25

Figure 2 graphically shows the ARL_1 performance of HWMA control charts using n=5, ARL_0=100. We observed that the efficiency of the HWMA control chart increase (i.e. ARL_1 values decrease) with the increase in shift from 10% to 40% respectively. The ARL_1values decrease for a fixed value of ARL_0 and λ's.

Table 3: ARL_1 values for HWMA Control Chart for n=5, ARL_o=200 and λ=0.15

Incontrol ARL0=200					
ARL1					
Shifts	10% increase	15% increase	20%increase	30% increase	40% increase
σ					
0.5	41.11	40.81	38.53	32.67	29.96
1	42.2	41.22	43.31	34.12	28.82
2	44.48	42.89	41.42	39.77	28.24
3	46.9	42.04	39.61	34.77	28.49
5	43.41	47.65	39.05	33.17	32.39
8	44.08	43.62	38.9	35.46	28.18
10	45.15	42.97	38.75	33.97	28.64

The table 3 describes the ARL_1 performance of proposed HWMA control chart by taking Rayleigh distribution using n=5, and ARL_0=200. We observed that the efficiency of the HWMA control chart increases (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively.
Figure 3 graphically shows the ARL_1 performance of HWMA control charts using n=5, ARL_0=200. We observed that the efficiency of the HWMA control chart increases (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively. The ARL_1values decrease as for a fixed value of ARL_0 and λ's.
The table 4 describes the ARL_1 performance of proposed HWMA control chart by taking Rayleigh distribution using n=5, and ARL_0=100. We observed that the efficiency of the HWMA control chart increases (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively.
Figure 4 graphically shows the ARL_1 performance of HWMA control charts using n=5, ARL_0=100. We observed that the efficiency of the HWMA control chart increases (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively. The ARL_1values decrease for a fixed value of ARL_0 and λ's.
The table 5 describes the ARL_1 performance of proposed HWMA control chart by taking Rayleigh distribution using n=5, and ARL_0=200. We observed that the efficiency of the HWMA control chart increases (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively.

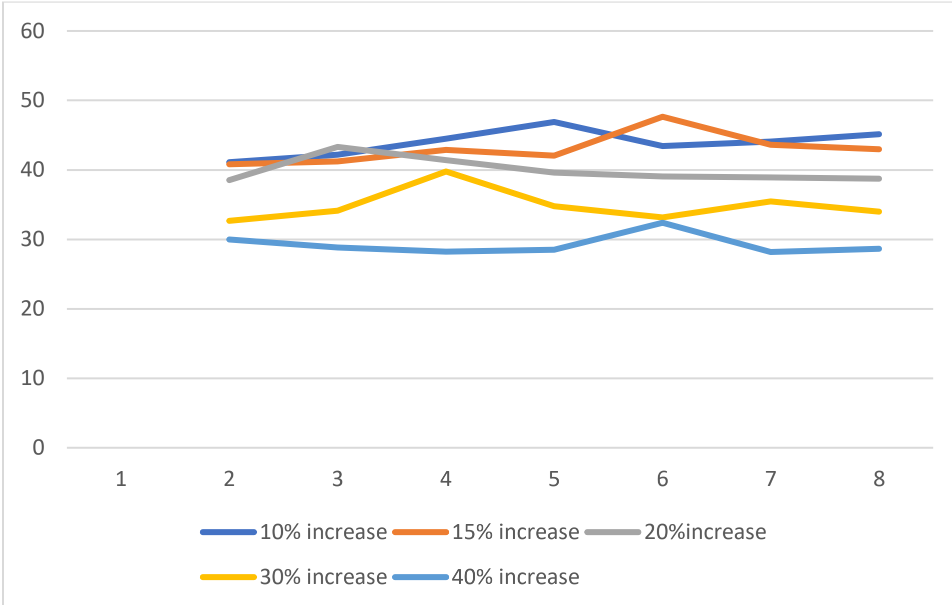

Figure 3: ARL_1 values for HWMA Control Chart for n=5, ARL_o=200 and λ=0.15

Table 4.: ARL_1 values for HWMA Control Chart for n=5, ARL_o=100 and λ=0.15

Incontrol ARL_0=100					
ARL_1					
Shifts	10% increase	15% increase	20%increase	30% increase	40% increase
σ					
0.5	25.6	21.33	19.03	20.74	17.3
1	37.76	16.69	25.97	13.91	12.66
2	20.99	18.43	22.1	13.76	19.94
3	23.19	15.88	23.23	17.7	14.4
5	19.23	17.91	18.68	19.5	19.18
8	22.08	13.63	17.93	13.97	13.59
10	18.66	13.3	24.96	18.55	14.4

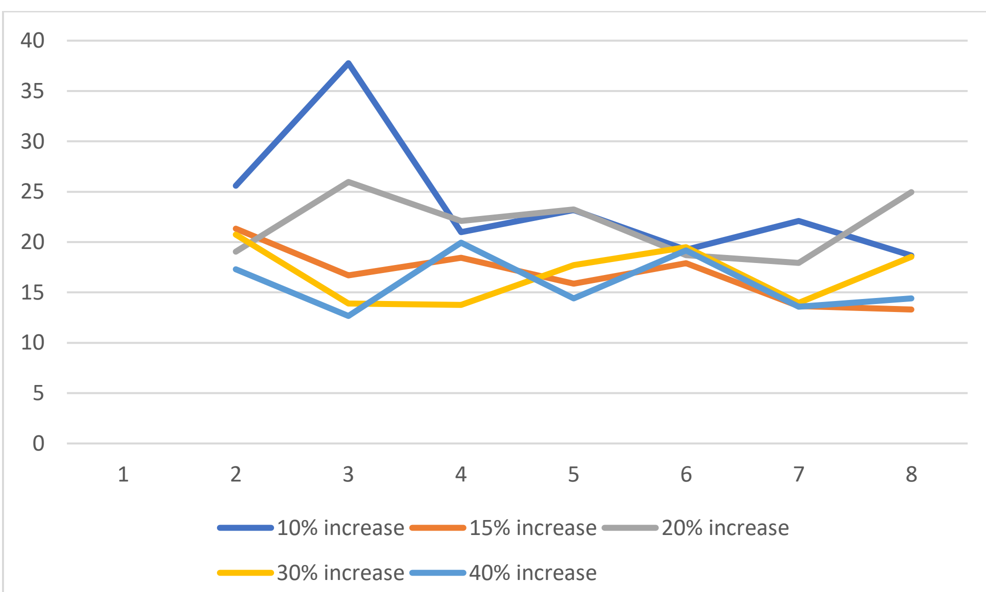

Figure 4: ARL_1 values for HWMA Control Chart for n=5 , ARL_o=100 and λ=0.15

Table 5: ARL_1 values for HWMA Control Chart for n=5, ARL_0=200 and λ=0.35

Incontrol ARL0=200					
ARL1					
Shifts	10% increase	15% increase	20%increase	30% increase	40% increase
σ					
0.5	49.52	45.98	40.3	31.28	21.27
1	53.76	42.57	40.26	30.2	21.25
2	51.72	42.49	42.7	34.25	20.94
3	43.1	44.6	40.67	29.13	23.03
5	45.01	41.47	41.27	28.95	20.37
8	51.14	50.81	41.29	35.64	19.92
10	48.68	43.85	40.75	31.18	22.92

Table 6: ARL_1 values for HWMA Control Chart for n=5, ARL_0=100 and λ=0.35

Incontrol ARL0=100					
ARL1					
Shifts	10% increase	15% increase	20% increase	30% increase	40% increase
σ					
0.5	35.39	32.84	32.01	24.79	18.45
1	28.98	31.93	32.79	25.86	18.7
2	38.74	32.54	32.06	24.43	15.87
3	33.71	31.76	30.82	22.43	18.78
5	29.7	31.63	29.57	25.39	17.91
8	34.06	30.1	28.23	24.25	19.12
10	38.03	35.13	28.12	21.28	18.27

The table 6 describes the ARL_1 performance of proposed HWMA control chart by taking Rayleigh distribution using n=5, and ARL_0=100. We observed that the efficiency of the HWMA control chart increases (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively.

Table 7: ARL_1 values for HWMA Control Chart for n=5, ARL_0=200 and λ=0.25

Incontrol ARL0=200					
ARL1					
Shifts	10% decrease	15% decrease	20%decrease	30% decrease	40% decrease
σ					
0.5	46.53(51.04)	44.68(51.96)	41.72(49.06)	39.72(46.32)	31.32(37.22)
1	46.32(54.38)	45.90(51.94)	41.28(51.73)	37.02(45.53)	31.59(39.28)
2	46.66(56.23)	43.23(52.66)	41.39(48.79)	39.34(44.72)	32.70(41.76)
3	52.37(57.69)	46.09(52.98)	41.87(47.98)	40.01(46.19)	33.86(42.48)
5	46.01(54.87)	50.71(55.44)	48.55(51.98)	37.92(44.92)	34.70(40.97)
8	45.70(50.48)	46.15(50.45)	47.67(53.29)	43.42(47.30)	36.27(38.43)
10	52.64(58.60)	46.97(53.96)	44.07(51.65)	41.65(46.35)	34.37(37.77)

The table 7 describes the ARL_1 performance of proposed HWMA control chart by taking Rayleigh distribution using n=5, and ARL_0=200. We observed that the efficiency of the HWMA control chart increases (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively.

The table 8 describes the ARL_1 performance of proposed HWMA control chart by taking Rayleigh distribution using n=5, and ARL_0=100. We observed that the efficiency of the HWMA control chart increases (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively.

The table 9 describes the ARL_1 performance of proposed EWMA control chart by taking Rayleigh distribution using n=5, and ARL_0=200. We observed that the efficiency of the EWMA control chart increases (i.e. ARL_1 values decreases) with the increase in Shift from 10% to 40% respectively.

Table 8: ARL_1 values for HWMA Control Chart for n=5, ARL_o=100 and λ=0.25

Incontrol ARL0=100					
ARL1					
Shifts	10% decrease	15% decrease	20%decrease	30% decrease	40% decrease
σ					
0.5	28.95(81.31)	25.63(74.57)	21.5(74.43)	22.15(66.67)	23.98(60.19)
1	24.63(73.42)	27.26(70.19)	22.38(77.80)	26.20(65.23)	20.99(59.64))
2	25.40(84.05)	18.84(82.17)	17.20(67.06)	16.29(65.87)	16.02(51.32)
3	34.63(85.48)	18.56(74.99)	21.11(70.07)	24.52(65.03)	21.70(50.55)
5	31.13(74.44)	20.38(80.55)	17.70(66.94)	21.85(66.88)	21.17(59.77)
8	33.91(79.51)	25.66(79.37)	22.20(69.48)	24.19(56.07)	23.44(54.75)
10	24.71(63.25)	29.31(82.39)	20.28(63.79)	17.09(67.14)	23.64(45.09)

Table 9: ARL_1 values for EWMA Control Chart for n=5, ARL_o=200 and λ=0.25

Incontrol ARL0=200, λ=0.25					
ARL1					
Shifts	10% increase	15% increase	20%increase	30% increase	40% increase
σ					
0.5	42.842	36.085	30.9	17.815	11.89
1	43.68	36.655	30.755	17.67	11.81
2	45.48	37.71	31.255	17.33	11.975
3	45.615	40.675	33.26	18.875	12.615
5	47.16	39.115	31.61	18.785	12.75
8	49.525	41.14	32.58	19	12.04
10	45.1	40.58	33.52	19.88	12.05

Table 10: Comparison of ARL_1 values for EWMA and HWMA Control Chart for n=5, ARL_o=200 and λ=0.35

In control ARL0=200, λ=0.35										
ARL1										
Shifts	10% increase		15% increase		20%increase		30% increase		40% increase	
Σ	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA
0.5	43.39	49.52	36.37	45.98	34.22	40.3	22.49	31.28	12.04	21.27
1	44.84	53.76	37.99	42.57	32.65	40.26	20.58	30.2	12.98	21.25
2	46.14	51.72	37.49	42.49	33.37	42.7	18.82	34.25	12.43	20.94
3	44.39	47.29	37.62	44.6	33.1	40.67	18.51	29.13	12.72	23.03
5	45.01	47.24	37.61	41.47	32.71	41.27	18.55	28.95	12.53	20.37
8	46.5	51.14	40.34	50.81	33	41.29	18.31	35.64	12.24	19.92
10	47.34	48.68	38.25	43.85	31.04	40.75	20.33	31.18	12.49	22.92

The table 10 describe the comparison of ARL_1 values of proposed HWMA control chart and EWMA control chart using n=5, and ARL_0=200. For 10% increase in shift, $\sigma = 0.5$ the ARL_1 value for HWMA chart is found to be 43.39 and for EWMA is found to be 49.52 which shows that HWMA chart out performs. Similarly, the same pattern of results observed σ from 0.5 – 10 and increase in shift from 10% – 40%. In comparison of the efficiency of the HWMA chart increases (i.e. ARL_1 values reduces) with the increase in Shift from 10% to 40% respectively. The HWMA performs better than the EWMA control charts.
The table 11 describe the comparison of ARL_1 values of proposed HWMA control chart and EWMA control chart using n=5, and ARL_0=100. For 10% increase in shift, $\sigma = 0.5$ the ARL_1 value for HWMA chart is found to be 35.39 and for EWMA is found to be 39.41 which shows that HWMA chart out performs. Similarly, the same pattern of results observed σ from 0.5 – 10 and increase in shift from 10% – 40%. In comparison of the efficiency of the HWMA chart increases (i.e. ARL_1 values reduces) with the increase in Shift from 10% to 40% respectively. The HWMA performs better than the EWMA control charts.
The table 12 describe the comparison of ARL_1 values of proposed HWMA control chart and EWMA control chart using n=5, and ARL_0=200. For 10% increase in shift, $\sigma = 0.5$ the ARL_1 value for HWMA chart is found to be 41.11 and for EWMA is found to be 42.9 which shows that HWMA chart out performs. Similarly, the same pattern of results observed σ from 0.5 – 10 and increase in shift from 10% – 40%. In comparison of the efficiency of the HWMA chart increases (i.e. ARL_1 values reduces) with the increase in Shift from 10% to 40% respectively. The HWMA performs better than the EWMA control charts.

Table 11: Comparison of ARL_1 values for EWMA and HWMA Control Chart for n=5, ARL_o=100 and λ=0.35

Incontrol ARL0=100, λ=0.35										
ARL1										
Shifts	10% increase		15% increase		20%increase		30% increase		40% increase	
Σ	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA
0.5	35.39	39.41	26.47	32.84	24.69	32.01	18.8	24.79	8.1	18.45
1	28.98	34.94	26.15	31.93	21.56	32.79	18.16	25.86	9.99	18.7
2	38.74	39.89	26.39	32.54	20.11	32.06	14.53	24.43	10.51	15.87
3	33.71	34.12	26.76	31.76	21.25	30.82	15.13	22.43	10.39	18.78
5	29.7	37.02	27.1	31.63	22.41	29.57	19.58	25.39	10.67	17.91
8	32.07	34.06	29.3	30.1	23.29	28.23	16.64	24.25	10.7	19.12
10	30.26	38.03	32.79	35.13	23.1	28.12	14.95	21.28	10.78	18.27

Table 12: Comparison of ARL_1 values for EWMA and HWMA Control Chart for n=5, ARL_o=200 and λ=0.15

Incontrol ARL0=200, λ=0.15										
ARL1										
Shifts	10% increase		15% increase		20%increase		30% increase		40% increase	
Σ	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA
0.5	41.11	42.9	40.81	34.47	25.24	38.53	13.79	32.67	11.66	29.96
1	42.2	43.31	41.22	34.63	25.79	43.31	15.32	34.12	11.53	28.82
2	44.04	44.48	42.89	36.64	27.87	41.42	15.25	39.77	11.14	28.24
3	44.25	46.9	42.04	37.03	27.5	39.61	15.79	34.77	10.43	28.49
5	43.41	46.51	47.65	36.69	27.76	39.05	15.99	33.17	10.75	32.39
8	44.08	46.44	43.62	38.01	27.44	38.9	15.63	35.46	11.69	28.18
10	45.09	45.15	42.97	37.4	27.79	38.75	16.64	33.97	11.68	28.64

Table 13: Comparison of ARL_1 values for EWMA and HWMA Control Chart for n=5, ARL_o=100 and λ=0.15

Incontrol ARL0=100, λ=0.15										
ARL1										
Shifts	10% increase		15% increase		20%increase		30% increase		40% increase	
Σ	EWMA	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA	HWMA	EWMA	HWMA
0.5	29.98	25.6	25.31	21.33	22.05	19.03	20.74	15.04	17.3	9.58
1	37.76	27.26	27.35	16.69	26.2	25.97	15.29	13.91	12.66	9.24
2	36.76	20.99	30.02	18.43	22.97	22.1	14.76	13.76	19.94	8.75
3	33.09	23.19	30.61	15.88	23.63	21.23	17.7	13.62	14.4	10.58
5	35.31	19.23	28.63	17.91	20.25	18.68	19.5	13.59	19.18	9.31
8	25.07	22.08	26.02	13.63	19.44	17.93	13.97	13.48	13.59	9.13
10	29.38	18.66	26.59	13.3	24.68	20.96	18.55	10.57	14.4	8.42

The table 13 describe the comparison of ARL_1 of proposed HWMA control chart and EWMA control chart using n=5, and ARL_0=100. For 10% increase in shift, $\sigma = 0.5$ the ARL_1 value for HWMA chart is found to be 25.5 and for EWMA is found to be 29.98 which shows that HWMA chart out performs. Similarly, the same pattern of results observed σ from 0.5 – 10 and increase in shift from 10% – 40%. In comparison of the efficiency of the HWMA chart increases (i.e. ARL_1 values reduces) with the increase in Shift from 10% to 40% respectively. The HWMA performs better than the EWMA control charts.

3.2.2. Application of Proposed HWMA Control Chart on Real Life Data

To study the application and execution of proposed HWMA control chart on real life data, we have taken a dataset of 30 average daily wind speeds (in kilometers/hour) for the month of November 2007 at Elanora Heights, a north-eastern suburb of Sydney, Australia. (Best et al. 2010). The data is given below

2.7	3.2	2.1	4.8	7.6	4.7	4.2	4.0	2.9	2.9
4.6	4.8	4.3	4.6	3.7	2.4	4.9	4.0	7.7	10.0
5.2	2.6	4.2	3.6	2.5	3.3	3.1	3.7	2.8	4.0

To check the distribution of data, we fit the data using *easyfit* software and found that the data follows Rayleigh distribution with a scale parameter σ = 3.327.
We generate a random sample of size 3 from the dataset of daily wind speed in which we took 30 in-control sample and 30 sample with shift in scale parameter. We introduced 20% decrease shift in scale parameter σ and calculated the ARL_1 which is equal to 8.36. This value indicates that the control chart need 8 sample points on average to detect an out-of-control signal when there is 20% decrease shift.

4. Conclusions and Recommendations

This paper provides a new control charting methodology for monitoring Rayleigh lifetimes. The proposal explains the homogeneously weighted moving average (HWMA) control chart. The proposed HWMA Chart is compared with the existing EWMA charting methodology. The ARL_1 criterion is used for the comparison of control charts efficiency. The HWMA control chart efficiency is compared to the Exponentially weighted moving average (EWMA) control charts with respect to the different shift size (i.e. 10%, 15%, 20%, 30% , 40% increase and decrease in shift). The sample size of size n= 5 (using simple random sampling) is used for the results calculations and elaborations. It is observed that with the increase in shift proposed HWMA chart shows more efficient results i.e. ARL_1 values decreases. In comparison to EWMA charts, it is found that the proposed HWMA chart for Rayleigh distribution outperforms the existing EWMA control chart. The minimum ARL_1 values are obtained for 40% increase and decrease in shifts. It is also observed that for moderate values of σ i.e. 1-5 the ARL_1 performance is consistent and when σ >5 the performance of the proposed chart decreases.
The proposed chart is developed for Rayleigh distributed lifetimes. The work can be further extended for other skewed distribution like Gamma, exponential, Weibull distributions.
The work can also be extended by developing a Double Homogeneously weighted moving average (DHWMA) control chart for Rayleigh distribution.

Appendix-A: Table Simulated Samples and ResultS

UNIFORM DIST					RAYLEIGH					MEAN	HWMA-ST.
0.267653	0.948767	0.437652	0.795525	0.380436	1.76494	5.451036	2.399237	3.984103	2.188012	3.157466	3.157466
0.198012	0.451799	0.063311	0.873144	0.495485	1.485467	2.451763	0.808728	4.5439	2.615641	2.3811	5.538565
0.957571	0.214406	0.033038	0.374976	0.130869	5.621324	1.55343	0.579625	2.16787	1.184318	2.221313	7.759879
0.648757	0.753464	0.858267	0.726638	0.6203	3.234619	3.741986	4.420193	3.601332	3.111872	3.622	11.38188
0.547563	0.7306	0.029845	0.069378	0.734431	2.816215	3.621543	0.550445	0.847949	3.641264	2.295483	13.67736
0.084488	0.53618	0.06852	0.17705	0.297665	0.93953	2.771748	0.842502	1.395921	1.879747	1.56589	15.24325
0.70681	0.719469	0.821988	0.271109	0.313511	3.502762	3.565208	4.154398	1.778289	1.939499	2.988031	18.23128
0.758994	0.267639	0.199349	0.575133	0.850808	3.772176	1.764885	1.491074	2.925711	4.361792	2.863128	21.09441
0.552609	0.806414	0.23259	0.3817	0.895345	2.836058	4.0522	1.627067	2.192671	4.750881	3.091775	24.18619
0.997366	0.870752	0.057519	0.104392	0.825572	7.706598	4.523302	0.769673	1.050009	4.178806	3.645678	27.83186
0.312162	0.162079	0.826692	0.825268	0.357745	1.934431	1.329782	4.186508	4.176722	2.104209	2.74633	30.57819
0.639841	0.579459	0.310738	0.799061	0.269727	3.195639	2.94315	1.929077	4.005936	1.772955	2.769351	33.34755
0.226971	0.588153	0.678421	0.878056	0.480752	1.604488	2.978428	3.368254	4.587149	2.560026	3.019669	36.36721
0.991931	0.617915	0.496611	0.4424	0.760028	6.942421	3.101794	2.61991	2.416843	3.777874	3.771769	40.13898
0.504207	0.497818	0.992256	0.454943	0.418513	2.648766	2.624484	6.971997	2.463464	2.328449	3.407432	43.54641
0.361082	0.960061	0.560078	0.89355	0.858143	2.11655	5.674851	2.865585	4.732944	4.4192	3.961826	47.50824
0.618456	0.112533	0.002072	0.043901	0.828911	3.104077	1.092629	0.144008	0.670025	4.201868	1.842521	49.35076
0.539092	0.513114	0.643304	0.461323	0.945145	2.783084	2.68277	3.210717	2.487244	5.388015	3.310366	52.66113
0.15766	0.812563	0.014391	0.857294	0.628503	1.309853	4.091834	0.380736	4.412444	3.146766	2.668327	55.32946
0.089881	0.592489	0.167419	0.190034	0.840141	0.970463	2.996143	1.353605	1.45177	4.281894	2.210775	57.54023
0.045086	0.784834	0.020759	0.030218	0.369248	0.679224	3.919622	0.458013	0.553933	2.146725	1.551503	59.09173
0.226718	0.670126	0.574915	0.778096	0.206179	1.60347	3.33023	2.924835	3.88009	1.519532	2.651632	61.74337
0.234904	0.283537	0.946452	0.789951	0.189433	1.636321	1.826003	5.410337	3.950209	1.449211	2.854416	64.59778
0.067033	0.069649	0.159669	0.816753	0.175956	0.832982	0.849664	1.318936	4.11937	1.391156	1.702422	66.3002
0.387598	0.624917	0.149575	0.908453	0.644392	2.214422	3.131468	1.272868	4.889688	3.215473	2.944784	69.24499

0.693466	0.444261	0.220666	0.743877	0.532436	3.438644	2.42375	1.578974	3.690662	2.757208	2.777848	72.02283
0.868248	0.692413	0.868594	0.569176	0.109653	4.502038	3.433654	4.504958	2.901819	1.077704	3.284035	75.30687
0.239514	0.043669	0.735508	0.532837	0.148688	1.654683	0.668216	3.646838	2.758763	1.268765	1.999453	77.30632
0.760774	0.594362	0.758587	0.457238	0.711159	3.781995	3.00382	3.769939	2.472012	3.524031	3.310359	80.61668
0.381752	0.782937	0.360872	0.478459	0.615222	2.192866	3.908413	2.115777	2.551406	3.090453	2.771783	83.38846
0.042588	0.349615	0.129594	0.065386	0.953029	0.659704	2.074104	1.178115	0.822322	5.530124	2.052874	85.44134
0.721949	0.703315	0.450675	0.091028	0.27676	3.577642	3.485804	2.447579	0.97694	1.800039	2.457601	87.89894
0.91814	0.524971	0.25656	0.534386	0.13822	5.002738	2.728334	1.721822	2.764776	1.219654	2.687465	90.5864
0.29909	0.458878	0.182834	0.146905	0.923206	1.885142	2.478125	1.42096	1.260493	5.066196	2.422183	93.00859
0.442683	0.411014	0.688133	0.398941	0.65649	2.417894	2.300766	3.413472	2.25624	3.268854	2.731445	95.74003
0.893405	0.408221	0.364987	0.706901	0.882937	4.731511	2.290464	2.130986	3.503207	4.631462	3.457526	99.19756
0.52187	0.9746	0.917668	0.791722	0.957215	2.716382	6.060526	4.996996	3.960909	5.613893	4.669741	103.8673
0.560007	0.886869	0.870727	0.23386	0.748548	2.865305	4.668203	4.523082	1.632148	3.71551	3.48085	107.3481
0.112261	0.905135	0.292407	0.786138	0.224599	1.091227	4.85314	1.859802	3.927369	1.594913	2.66529	110.0134
0.925141	0.265613	0.551236	0.959546	0.682931	5.091318	1.757039	2.830652	5.663561	3.389155	3.746345	113.7598
0.877053	0.559834	0.226323	0.393631	0.283629	4.57821	2.864618	1.601878	2.236662	1.826356	2.621545	116.3813
0.809859	0.948636	0.996824	0.514056	0.669832	4.074294	5.44868	7.584275	2.686376	3.328892	4.624503	121.0058
0.401266	0.918872	0.48654	0.947274	0.622171	2.264815	5.011713	2.581828	5.424611	3.119796	3.680553	124.6864
0.145343	0.252734	0.480365	0.910569	0.925586	1.253214	1.706849	2.55857	4.913536	5.097173	3.105869	127.7923
0.51097	0.972339	0.129542	0.893388	0.41399	2.674569	5.989774	1.177863	4.731339	2.31175	3.377059	131.1693
0.660173	0.407619	0.001886	0.26989	0.7929	3.285298	2.288242	0.137388	1.773586	3.968063	2.290516	133.4598
0.205896	0.828186	0.157613	0.743535	0.601043	1.518357	4.196836	1.30964	3.688853	3.031341	2.749005	136.2088
0.331402	0.386504	0.032681	0.983167	0.562905	2.00642	2.210389	0.576431	6.390933	2.87681	2.812197	139.021
0.457046	0.179586	0.321599	0.582447	0.059432	2.471296	1.406934	1.969813	2.955238	0.782757	1.917208	140.9382
0.383342	0.289396	0.569169	0.03471	0.776164	2.198728	1.848351	2.901793	0.594366	3.868906	2.282429	143.2207
0.124225	0.910454	0.024733	0.772928	0.342578	1.151719	4.912233	0.500442	3.850311	2.047998	2.49254	145.7132
0.669742	0.786803	0.694441	0.706499	0.979952	3.328485	3.931332	3.443272	3.501249	6.25269	4.091406	149.8046
0.550382	0.881991	0.995291	0.472601	0.565711	2.827291	4.62276	7.319992	2.529424	2.887983	4.03749	153.8421
0.400579	0.674753	0.162316	0.503052	0.7886	2.26228	3.351375	1.330842	2.644371	3.942086	2.706191	156.5483
0.936353	0.277682	0.358262	0.230677	0.43909	5.248246	1.803579	2.106122	1.619396	2.404568	2.636382	159.1847
0.315556	0.279618	0.773527	0.50577	0.237518	1.947172	1.811004	3.853736	2.654721	1.646744	2.382675	161.5674
0.673332	0.475239	0.26856	0.80238	0.326579	3.344862	2.539314	1.768445	4.026675	1.988429	2.733545	164.3009
0.566819	0.029201	0.803045	0.149194	0.13943	2.892402	0.544392	4.030857	1.271108	1.225397	1.992831	166.2937
0.787743	0.652796	0.579165	0.567044	0.828219	3.936951	3.25245	2.941961	2.893301	4.197064	3.444345	169.7381
0.735807	0.686101	0.78536	0.445988	0.027486	3.648388	3.403947	3.922748	2.430164	0.527927	2.786635	172.5247
0.768046	0.814432	0.31259	0.217817	0.697967	3.822585	4.104063	1.936037	1.567373	3.460086	2.978029	175.5027
0.112654	0.740557	0.388219	0.451569	0.749176	1.093256	3.673168	2.21671	2.450907	3.718876	2.630583	178.1333
0.810843	0.84033	0.328396	0.183545	0.224922	4.080661	4.28328	1.995209	1.424019	1.59622	2.675878	180.8092
0.031541	0.645001	0.03614	0.045369	0.50277	0.566121	3.218138	0.606702	0.681395	2.643298	1.543131	182.3523
0.974439	0.448752	0.58348	0.892001	0.382132	6.055328	2.440433	2.959427	4.717662	2.194267	3.673423	186.0258
0.98722	0.408294	0.503055	0.297204	0.980399	6.602933	2.290731	2.644383	1.878001	6.270696	3.937349	189.9631
0.303909	0.77115	0.59757	0.362618	0.444396	1.90335	3.840168	3.017009	2.122229	2.42425	2.661401	192.6245
0.350593	0.314151	0.568195	0.72844	0.39858	2.07773	1.9419	2.897896	3.6105	2.254909	2.556587	195.1811
0.38592	0.582262	0.197292	0.083173	0.891206	2.208234	2.954491	1.482443	0.931859	4.709878	2.457381	197.6385

0.585164	0.515284	0.981237	0.427059	0.628645	2.966263	2.691082	6.305456	2.360025	3.147373	3.49404	201.1325
0.325162	0.119014	0.975131	0.779148	0.791463	1.983135	1.12567	6.077933	3.886207	3.959344	3.406458	204.539
0.547563	0.7306	0.029845	0.069378	0.734431	2.816215	3.621543	0.550445	0.847949	3.641264	2.295483	13.67736
0.084488	0.53618	0.06852	0.17705	0.297665	0.93953	2.771748	0.842502	1.395921	1.879747	1.56589	15.24325
0.70681	0.719469	0.821988	0.271109	0.313511	3.502762	3.565208	4.154398	1.778289	1.939499	2.988031	18.23128
0.758994	0.267639	0.199349	0.575133	0.850808	3.772176	1.764885	1.491074	2.925711	4.361792	2.863128	21.09441
0.552609	0.806414	0.23259	0.3817	0.895345	2.836058	4.0522	1.627067	2.192671	4.750881	3.091775	24.18619
0.997366	0.870752	0.057519	0.104392	0.825572	7.706598	4.523302	0.769673	1.050009	4.178806	3.645678	27.83186
0.312162	0.162079	0.826692	0.825268	0.357745	1.934431	1.329782	4.186508	4.176722	2.104209	2.74633	30.57819
0.639841	0.579459	0.310738	0.799061	0.269727	3.195639	2.94315	1.929077	4.005936	1.772955	2.769351	33.34755
0.226971	0.588153	0.678421	0.878056	0.480752	1.604488	2.978428	3.368254	4.587149	2.560026	3.019669	36.36721
0.991931	0.617915	0.496611	0.4424	0.760028	6.942421	3.101794	2.61991	2.416843	3.777874	3.771769	40.13898
0.504207	0.497818	0.992256	0.454943	0.418513	2.648766	2.624484	6.971997	2.463464	2.328449	3.407432	43.54641
0.361082	0.960061	0.560078	0.89355	0.858143	2.11655	5.674851	2.865585	4.732944	4.4192	3.961826	47.50824
0.618456	0.112533	0.002072	0.043901	0.828911	3.104077	1.092629	0.144008	0.670025	4.201868	1.842521	49.35076
0.539092	0.513114	0.643304	0.461323	0.945145	2.783084	2.68277	3.210717	2.487244	5.388015	3.310366	52.66113
0.15766	0.812563	0.014391	0.857294	0.628503	1.309853	4.091834	0.380736	4.412444	3.146766	2.668327	55.32946
0.089881	0.592489	0.167419	0.190034	0.840141	0.970463	2.996143	1.353605	1.45177	4.281894	2.210775	57.54023
0.045086	0.784834	0.020759	0.030218	0.369248	0.679224	3.919622	0.458013	0.553933	2.146725	1.551503	59.09173
0.226718	0.670126	0.574915	0.778096	0.206179	1.60347	3.33023	2.924835	3.88009	1.519532	2.651632	61.74337
0.234904	0.283537	0.946452	0.789951	0.189433	1.636321	1.826003	5.410337	3.950209	1.449211	2.854416	64.59778
0.067033	0.069649	0.159669	0.816753	0.175956	0.832982	0.849664	1.318936	4.11937	1.391156	1.702422	66.3002
0.387598	0.624917	0.149575	0.908453	0.644392	2.214422	3.131468	1.272868	4.889688	3.215473	2.944784	69.24499
0.693466	0.444261	0.220666	0.743877	0.532436	3.438644	2.42375	1.578974	3.690662	2.757208	2.777848	72.02283
0.868248	0.692413	0.868594	0.569176	0.109653	4.502038	3.433654	4.504958	2.901819	1.077704	3.284035	75.30687
0.239514	0.043669	0.735508	0.532837	0.148688	1.654683	0.668216	3.646838	2.758763	1.268765	1.999453	77.30632
0.760774	0.594362	0.758587	0.457238	0.711159	3.781995	3.00382	3.769939	2.472012	3.524031	3.310359	80.61668
0.381752	0.782937	0.360872	0.478459	0.615222	2.192866	3.908413	2.115777	2.551406	3.090453	2.771783	83.38846
0.042588	0.349615	0.129594	0.065386	0.953029	0.659704	2.074104	1.178115	0.822322	5.530124	2.052874	85.44134
0.721949	0.703315	0.450675	0.091028	0.27676	3.577642	3.485804	2.447579	0.97694	1.800039	2.457601	87.89894
0.91814	0.524971	0.25656	0.534386	0.13822	5.002738	2.728334	1.721822	2.764776	1.219654	2.687465	90.5864
0.29909	0.458878	0.182834	0.146905	0.923206	1.885142	2.478125	1.42096	1.260493	5.066196	2.422183	93.00859
0.442683	0.411014	0.688133	0.398941	0.65649	2.417894	2.300766	3.413472	2.25624	3.268854	2.731445	95.74003
0.893405	0.408221	0.364987	0.706901	0.882937	4.731511	2.290464	2.130986	3.503207	4.631462	3.457526	99.19756
0.52187	0.9746	0.917668	0.791722	0.957215	2.716382	6.060526	4.996996	3.960909	5.613893	4.669741	103.8673
0.560007	0.886869	0.870727	0.23386	0.748548	2.865305	4.668203	4.523082	1.632148	3.71551	3.48085	107.3481
0.112261	0.905135	0.292407	0.786138	0.224599	1.091227	4.85314	1.859802	3.927369	1.594913	2.66529	110.0134
0.925141	0.265613	0.551236	0.959546	0.682931	5.091318	1.757039	2.830652	5.663561	3.389155	3.746345	113.7598
0.877053	0.559834	0.226323	0.393631	0.283629	4.57821	2.864618	1.601878	2.236662	1.826356	2.621545	116.3813
0.809859	0.948636	0.996824	0.514056	0.669832	4.074294	5.44868	7.584275	2.686376	3.328892	4.624503	121.0058
0.401266	0.918872	0.48654	0.947274	0.622171	2.264815	5.011713	2.581828	5.424611	3.119796	3.680553	124.6864
0.145343	0.252734	0.480365	0.910569	0.925586	1.253214	1.706849	2.55857	4.913536	5.097173	3.105869	127.7923
0.51097	0.972339	0.129542	0.893388	0.41399	2.674569	5.989774	1.177863	4.731339	2.31175	3.377059	131.1693
0.660173	0.407619	0.001886	0.26989	0.7929	3.285298	2.288242	0.137388	1.773586	3.968063	2.290516	133.4598

0.205896	0.828186	0.157613	0.743535	0.601043	1.518357	4.196836	1.30964	3.688853	3.031341	2.749005	136.2088
0.331402	0.386504	0.032681	0.983167	0.562905	2.00642	2.210389	0.576431	6.390933	2.87681	2.812197	139.021
0.457046	0.179586	0.321599	0.582447	0.059432	2.471296	1.406934	1.969813	2.955238	0.782757	1.917208	140.9382
0.383342	0.289396	0.569169	0.03471	0.776164	2.198728	1.848351	2.901793	0.594366	3.868906	2.282429	143.2207
0.124225	0.910454	0.024733	0.772928	0.342578	1.151719	4.912233	0.500442	3.850311	2.047998	2.49254	145.7132
0.669742	0.786803	0.694441	0.706499	0.979952	3.328485	3.931332	3.443272	3.501249	6.25269	4.091406	149.8046
0.550382	0.881991	0.995291	0.472601	0.565711	2.827291	4.62276	7.319992	2.529424	2.887983	4.03749	153.8421
0.400579	0.674753	0.162316	0.503052	0.7886	2.26228	3.351375	1.330842	2.644371	3.942086	2.706191	156.5483
0.936353	0.277682	0.358262	0.230677	0.43909	5.248246	1.803579	2.106122	1.619396	2.404568	2.636382	159.1847
0.315556	0.279618	0.773527	0.50577	0.237518	1.947172	1.811004	3.853736	2.654721	1.646744	2.382675	161.5674
0.673332	0.475239	0.26856	0.80238	0.326579	3.344862	2.539314	1.768445	4.026675	1.988429	2.733545	164.3009
0.566819	0.029201	0.803045	0.149194	0.13943	2.892402	0.544392	4.030857	1.271108	1.225397	1.992831	166.2937
0.787743	0.652796	0.579165	0.567044	0.828219	3.936951	3.25245	2.941961	2.893301	4.197064	3.444345	169.7381
0.735807	0.686101	0.78536	0.445988	0.027486	3.648388	3.403947	3.922748	2.430164	0.527927	2.786635	172.5247
0.768046	0.814432	0.31259	0.217817	0.697967	3.822585	4.104063	1.936037	1.567373	3.460086	2.978029	175.5027
0.112654	0.740557	0.388219	0.451569	0.749176	1.093256	3.673168	2.21671	2.450907	3.718876	2.630583	178.1333
0.810843	0.84033	0.328396	0.183545	0.224922	4.080661	4.28328	1.995209	1.424019	1.59622	2.675878	180.8092
0.031541	0.645001	0.03614	0.045369	0.50277	0.566121	3.218138	0.606702	0.681395	2.643298	1.543131	182.3523
0.974439	0.448752	0.58348	0.892001	0.382132	6.055328	2.440433	2.959427	4.717662	2.194267	3.673423	186.0258
0.98722	0.408294	0.503055	0.297204	0.980399	6.602933	2.290731	2.644383	1.878001	6.270696	3.937349	189.9631
0.303909	0.77115	0.59757	0.362618	0.444396	1.90335	3.840168	3.017009	2.122229	2.42425	2.661401	192.6245
0.350593	0.314151	0.568195	0.72844	0.39858	2.07773	1.9419	2.897896	3.6105	2.254909	2.556587	195.1811
0.38592	0.582262	0.197292	0.083173	0.891206	2.208234	2.954491	1.482443	0.931859	4.709878	2.457381	197.6385
0.585164	0.515284	0.981237	0.427059	0.628645	2.966263	2.691082	6.305456	2.360025	3.147373	3.49404	201.1325
0.147273	0.725988	0.172304	0.142253	0.925226	1.262206	3.598033	1.37517	1.238735	5.092434	2.513316	207.0523
0.045419	0.396339	0.528299	0.440672	0.564508	0.681786	2.246646	2.741186	2.410434	2.883191	2.192649	209.2449
0.701224	0.992416	0.033922	0.389347	0.293098	3.475718	6.986884	0.587461	2.22087	1.862427	3.026672	212.2716
0.920806	0.772909	0.198191	0.685384	0.610916	5.035724	3.850199	1.486219	3.400591	3.072393	3.369025	215.6406
0.187236	0.461623	0.942304	0.802243	0.487894	1.439843	2.488366	5.340943	4.025813	2.586937	3.17638	218.817
0.211913	0.057713	0.380788	0.642499	0.642732	1.543201	0.771006	2.18931	3.207208	3.208221	2.183789	221.0008
0.490479	0.375073	0.946445	0.083163	0.542706	2.596696	2.168224	5.410224	0.931805	2.797192	2.780828	223.7816
0.639961	0.918824	0.513976	0.271949	0.78957	3.19616	5.011121	2.686072	1.78153	3.947914	3.324559	227.1062
0.78654	0.277938	0.613942	0.445419	0.516339	3.929763	1.804561	3.085075	2.428049	2.695127	2.788515	229.8947
0.331979	0.436291	0.351346	0.2656	0.797911	2.008571	2.394195	2.08052	1.756991	3.998806	2.447817	232.3425
0.111762	0.450642	0.389337	0.641523	0.595297	1.088651	2.447457	2.220835	3.202955	3.007659	2.393511	234.736
0.389022	0.341063	0.492645	0.802348	0.80258	2.219673	2.042371	2.604888	4.02647	4.027928	2.984266	237.7203
0.204575	0.103531	0.645733	0.499079	0.395474	1.512872	1.045426	3.221344	2.629273	2.243458	2.130474	239.8508
0.842562	0.409461	0.297681	0.665977	0.708492	4.299676	2.295036	1.879807	3.311411	3.510967	3.059379	242.9101
0.174126	0.376722	0.762366	0.175909	0.900089	1.383159	2.174311	3.790811	1.390951	4.799454	2.707737	245.6179
0.4726	0.437003	0.822258	0.099824	0.178302	2.52942	2.396834	4.156228	1.025498	1.401364	2.301869	247.9198
0.459174	0.30398	0.055246	0.365261	0.441918	2.479228	1.903618	0.753863	2.131998	2.415055	1.936753	249.8565
0.632511	0.462599	0.80675	0.78342	0.72293	3.163955	2.492009	4.054344	3.911259	3.582577	3.440829	253.2973
0.607179	0.87252	0.577212	0.934602	0.511025	3.056797	4.538501	2.934082	5.222322	2.67478	3.685296	256.9826
0.22662	0.678441	0.494093	0.474229	0.482517	1.603073	3.368344	2.610369	2.535527	2.56667	2.536797	259.5194

0.114439	0.267281	0.497887	0.02344	0.02489	1.102422	1.763499	2.624749	0.487024	0.50205	1.295949	260.8154
0.784059	0.874605	0.194702	0.3257	0.239887	3.915035	4.556633	1.471539	1.985147	1.656164	2.716903	263.5323
0.838872	0.487092	0.264274	0.041298	0.350308	4.272654	2.58391	1.751848	0.649421	2.076672	2.266901	265.7992
0.619123	0.113044	0.347643	0.540793	0.733295	3.106893	1.09526	2.066792	2.789722	3.6354	2.538813	268.338
0.010765	0.075673	0.98246	0.100437	0.673009	0.328986	0.887068	6.358655	1.028818	3.343385	2.389382	270.7274
0.135153	0.055593	0.841057	0.557554	0.991774	1.205002	0.756294	4.288602	2.855584	6.928539	3.206804	273.9342
0.329296	0.080261	0.477589	0.877785	0.966378	1.998567	0.914686	2.548137	4.58473	5.82459	3.174142	277.1083
0.22385	0.433975	0.873292	0.868058	0.80699	1.591885	2.385618	4.545181	4.500437	4.055873	3.415799	280.5241
0.102841	0.021899	0.161737	0.543099	0.41967	1.041738	0.470556	1.328246	2.798729	2.332719	1.594397	282.1185
0.276774	0.972045	0.815707	0.686406	0.41103	1.800091	5.980932	4.112453	3.405374	2.300826	3.519935	285.6385
0.404586	0.031968	0.672024	0.109724	0.704069	2.277057	0.569997	3.338884	1.078071	3.489455	2.150693	287.7891
0.660344	0.045188	0.96672	0.79407	0.918185	3.286063	0.680009	5.833362	3.975199	5.003294	3.755585	291.5447
0.201567	0.567899	0.138428	0.258366	0.289993	1.500349	2.896713	1.220642	1.728869	1.850624	1.83944	293.3842
0.647116	0.626162	0.878632	0.075924	0.69871	3.227406	3.136771	4.592306	0.888598	3.463642	3.061745	296.4459
0.088527	0.282908	0.002007	0.86369	0.983554	0.962772	1.8236	0.141748	4.464109	6.409108	2.760268	299.2062
0.712352	0.059633	0.846959	0.03477	0.184703	3.529897	0.784125	4.332494	0.594886	1.428996	2.13408	301.3403
0.316626	0.150022	0.544666	0.423106	0.443536	1.951187	1.274929	2.804858	2.345414	2.421059	2.159489	303.4998
0.419728	0.498622	0.929028	0.806642	0.261427	2.332934	2.627537	5.143411	4.053652	1.740792	3.179665	306.6794
0.957034	0.185371	0.173318	0.221376	0.863017	5.610127	1.431859	1.379621	1.581855	4.458586	2.89241	309.5718
0.781516	0.642808	0.573891	0.391854	0.835061	3.900052	3.208552	2.92072	2.230113	4.245208	3.300929	312.8728
0.713282	0.756762	0.216518	0.335985	0.50253	3.534481	3.759942	1.562071	2.023488	2.642387	2.704474	315.5772
0.674396	0.3483	0.286033	0.820271	0.960252	3.349737	2.069228	1.835535	4.142831	5.679087	3.415283	318.9925
0.513957	0.764929	0.748092	0.673912	0.807208	2.685998	3.805083	3.713071	3.347517	4.057268	3.521787	322.5143
0.278621	0.509002	0.756749	0.155787	0.073909	1.807181	2.66705	3.75987	1.301348	0.876256	2.082341	324.5966
0.030653	0.024289	0.426884	0.116451	0.740055	0.557962	0.495871	2.359379	1.112693	3.670537	1.639288	326.2359
0.111013	0.091593	0.391107	0.726289	0.870311	1.084771	0.980118	2.227357	3.599559	4.519529	2.482267	328.7182
0.624546	0.297792	0.611764	0.471507	0.246609	3.129889	1.880227	3.075941	2.525325	1.682768	2.45883	331.177
0.57328	0.860307	0.218149	0.87149	0.197681	2.918267	4.436566	1.56873	4.529621	1.484081	2.987453	334.1645
0.087272	0.765028	0.954815	0.659378	0.426093	0.955599	3.805639	5.565065	3.281741	2.356456	3.1929	337.3574
0.451479	0.069781	0.207088	0.537985	0.348364	2.450571	0.850501	1.523296	2.778773	2.069465	1.934521	339.2919
0.70681	0.719469	0.821988	0.271109	0.313511	3.502762	3.565208	4.154398	1.778289	1.939499	2.988031	18.23128
0.758994	0.267639	0.199349	0.575133	0.850808	3.772176	1.764885	1.491074	2.925711	4.361792	2.863128	21.09441
0.552609	0.806414	0.23259	0.3817	0.895345	2.836058	4.0522	1.627067	2.192671	4.750881	3.091775	24.18619
0.997366	0.870752	0.057519	0.104392	0.825572	7.706598	4.523302	0.769673	1.050009	4.178806	3.645678	27.83186
0.312162	0.162079	0.826692	0.825268	0.357745	1.934431	1.329782	4.186508	4.176722	2.104209	2.74633	30.57819
0.639841	0.579459	0.310738	0.799061	0.269727	3.195639	2.94315	1.929077	4.005936	1.772955	2.769351	33.34755
0.226971	0.588153	0.678421	0.878056	0.480752	1.604488	2.978428	3.368254	4.587149	2.560026	3.019669	36.36721
0.991931	0.617915	0.496611	0.4424	0.760028	6.942421	3.101794	2.61991	2.416843	3.777874	3.771769	40.13898
0.504207	0.497818	0.992256	0.454943	0.418513	2.648766	2.624484	6.971997	2.463464	2.328449	3.407432	43.54641
0.361082	0.960061	0.560078	0.89355	0.858143	2.11655	5.674851	2.865585	4.732944	4.4192	3.961826	47.50824
0.618456	0.112533	0.002072	0.043901	0.828911	3.104077	1.092629	0.144008	0.670025	4.201868	1.842521	49.35076
0.539092	0.513114	0.643304	0.461323	0.945145	2.783084	2.68277	3.210717	2.487244	5.388015	3.310366	52.66113
0.15766	0.812563	0.014391	0.857294	0.628503	1.309853	4.091834	0.380736	4.412444	3.146766	2.668327	55.32946
0.089881	0.592489	0.167419	0.190034	0.840141	0.970463	2.996143	1.353605	1.45177	4.281894	2.210775	57.54023

0.045086	0.784834	0.020759	0.030218	0.369248	0.679224	3.919622	0.458013	0.553933	2.146725	1.551503	59.09173
0.226718	0.670126	0.574915	0.778096	0.206179	1.60347	3.33023	2.924835	3.88009	1.519532	2.651632	61.74337
0.234904	0.283537	0.946452	0.789951	0.189433	1.636321	1.826003	5.410337	3.950209	1.449211	2.854416	64.59778
0.067033	0.069649	0.159669	0.816753	0.175956	0.832982	0.849664	1.318936	4.11937	1.391156	1.702422	66.3002
0.387598	0.624917	0.149575	0.908453	0.644392	2.214422	3.131468	1.272868	4.889688	3.215473	2.944784	69.24499
0.693466	0.444261	0.220666	0.743877	0.532436	3.438644	2.42375	1.578974	3.690662	2.757208	2.777848	72.02283
0.868248	0.692413	0.868594	0.569176	0.109653	4.502038	3.433654	4.504958	2.901819	1.077704	3.284035	75.30687
0.239514	0.043669	0.735508	0.532837	0.148688	1.654683	0.668216	3.646838	2.758763	1.268765	1.999453	77.30632
0.760774	0.594362	0.758587	0.457238	0.711159	3.781995	3.00382	3.769939	2.472012	3.524031	3.310359	80.61668
0.381752	0.782937	0.360872	0.478459	0.615222	2.192866	3.908413	2.115777	2.551406	3.090453	2.771783	83.38846
0.042588	0.349615	0.129594	0.065386	0.953029	0.659704	2.074104	1.178115	0.822322	5.530124	2.052874	85.44134
0.721949	0.703315	0.450675	0.091028	0.27676	3.577642	3.485804	2.447579	0.97694	1.800039	2.457601	87.89894
0.91814	0.524971	0.25656	0.534386	0.13822	5.002738	2.728334	1.721822	2.764776	1.219654	2.687465	90.5864
0.29909	0.458878	0.182834	0.146905	0.923206	1.885142	2.478125	1.42096	1.260493	5.066196	2.422183	93.00859
0.442683	0.411014	0.688133	0.398941	0.65649	2.417894	2.300766	3.413472	2.25624	3.268854	2.731445	95.74003
0.893405	0.408221	0.364987	0.706901	0.882937	4.731511	2.290464	2.130986	3.503207	4.631462	3.457526	99.19756
0.52187	0.9746	0.917668	0.791722	0.957215	2.716382	6.060526	4.996996	3.960909	5.613893	4.669741	103.8673
0.560007	0.886869	0.870727	0.23386	0.748548	2.865305	4.668203	4.523082	1.632148	3.71551	3.48085	107.3481
0.112261	0.905135	0.292407	0.786138	0.224599	1.091227	4.85314	1.859802	3.927369	1.594913	2.66529	110.0134
0.925141	0.265613	0.551236	0.959546	0.682931	5.091318	1.757039	2.830652	5.663561	3.389155	3.746345	113.7598
0.877053	0.559834	0.226323	0.393631	0.283629	4.57821	2.864618	1.601878	2.236662	1.826356	2.621545	116.3813
0.809859	0.948636	0.996824	0.514056	0.669832	4.074294	5.44868	7.584275	2.686376	3.328892	4.624503	121.0058
0.401266	0.918872	0.48654	0.947274	0.622171	2.264815	5.011713	2.581828	5.424611	3.119796	3.680553	124.6864
0.145343	0.252734	0.480365	0.910569	0.925586	1.253214	1.706849	2.55857	4.913536	5.097173	3.105869	127.7923
0.51097	0.972339	0.129542	0.893388	0.41399	2.674569	5.989774	1.177863	4.731339	2.31175	3.377059	131.1693
0.660173	0.407619	0.001886	0.26989	0.7929	3.285298	2.288242	0.137388	1.773586	3.968063	2.290516	133.4598
0.205896	0.828186	0.157613	0.743535	0.601043	1.518357	4.196836	1.30964	3.688853	3.031341	2.749005	136.2088
0.331402	0.386504	0.032681	0.983167	0.562905	2.00642	2.210389	0.576431	6.390933	2.87681	2.812197	139.021
0.457046	0.179586	0.321599	0.582447	0.059432	2.471296	1.406934	1.969813	2.955238	0.782757	1.917208	140.9382
0.383342	0.289396	0.569169	0.03471	0.776164	2.198728	1.848351	2.901793	0.594366	3.868906	2.282429	143.2207
0.124225	0.910454	0.024733	0.772928	0.342578	1.151719	4.912233	0.500442	3.850311	2.047998	2.49254	145.7132
0.669742	0.786803	0.694441	0.706499	0.979952	3.328485	3.931332	3.443272	3.501249	6.25269	4.091406	149.8046
0.550382	0.881991	0.995291	0.472601	0.565711	2.827291	4.62276	7.319992	2.529424	2.887983	4.03749	153.8421
0.400579	0.674753	0.162316	0.503052	0.7886	2.26228	3.351375	1.330842	2.644371	3.942086	2.706191	156.5483
0.936353	0.277682	0.358262	0.230677	0.43909	5.248246	1.803579	2.106122	1.619396	2.404568	2.636382	159.1847
0.315556	0.279618	0.773527	0.50577	0.237518	1.947172	1.811004	3.853736	2.654721	1.646744	2.382675	161.5674
0.673332	0.475239	0.26856	0.80238	0.326579	3.344862	2.539314	1.768445	4.026675	1.988429	2.733545	164.3009
0.566819	0.029201	0.803045	0.149194	0.13943	2.892402	0.544392	4.030857	1.271108	1.225397	1.992831	166.2937
0.787743	0.652796	0.579165	0.567044	0.828219	3.936951	3.25245	2.941961	2.893301	4.197064	3.444345	169.7381
0.735807	0.686101	0.78536	0.445988	0.027486	3.648388	3.403947	3.922748	2.430164	0.527927	2.786635	172.5247
0.768046	0.814432	0.31259	0.217817	0.697967	3.822585	4.104063	1.936037	1.567373	3.460086	2.978029	175.5027
0.112654	0.740557	0.388219	0.451569	0.749176	1.093256	3.673168	2.21671	2.450907	3.718876	2.630583	178.1333
0.810843	0.84033	0.328396	0.183545	0.224922	4.080661	4.28328	1.995209	1.424019	1.59622	2.675878	180.8092
0.031541	0.645001	0.03614	0.045369	0.50277	0.566121	3.218138	0.606702	0.681395	2.643298	1.543131	182.3523

0.974439	0.448752	0.58348	0.892001	0.382132	6.055328	2.440433	2.959427	4.717662	2.194267	3.673423	186.0258
0.98722	0.408294	0.503055	0.297204	0.980399	6.602933	2.290731	2.644383	1.878001	6.270696	3.937349	189.9631
0.303909	0.77115	0.59757	0.362618	0.444396	1.90335	3.840168	3.017009	2.122229	2.42425	2.661401	192.6245
0.350593	0.314151	0.568195	0.72844	0.39858	2.07773	1.9419	2.897896	3.6105	2.254909	2.556587	195.1811
0.38592	0.582262	0.197292	0.083173	0.891206	2.208234	2.954491	1.482443	0.931859	4.709878	2.457381	197.6385
0.585164	0.515284	0.981237	0.427059	0.628645	2.966263	2.691082	6.305456	2.360025	3.147373	3.49404	201.1325
0.325162	0.119014	0.975131	0.779148	0.791463	1.983135	1.12567	6.077933	3.886207	3.959344	3.406458	204.539
0.147273	0.725988	0.172304	0.142253	0.925226	1.262206	3.598033	1.37517	1.238735	5.092434	2.513316	207.0523
0.045419	0.396339	0.528299	0.440672	0.564508	0.681786	2.246646	2.741186	2.410434	2.883191	2.192649	209.2449
0.701224	0.992416	0.033922	0.389347	0.293098	3.475718	6.986884	0.587461	2.22087	1.862427	3.026672	212.2716
0.920806	0.772909	0.198191	0.685384	0.610916	5.035724	3.850199	1.486219	3.400591	3.072393	3.369025	215.6406
0.187236	0.461623	0.942304	0.802243	0.487894	1.439843	2.488366	5.340943	4.025813	2.586937	3.17638	218.817
0.211913	0.057713	0.380788	0.642499	0.642732	1.543201	0.771006	2.18931	3.207208	3.208221	2.183789	221.0008
0.490479	0.375073	0.946445	0.083163	0.542706	2.596696	2.168224	5.410224	0.931805	2.797192	2.780828	223.7816
0.639961	0.918824	0.513976	0.271949	0.78957	3.19616	5.011121	2.686072	1.78153	3.947914	3.324559	227.1062
0.78654	0.277938	0.613942	0.445419	0.516339	3.929763	1.804561	3.085075	2.428049	2.695127	2.788515	229.8947
0.331979	0.436291	0.351346	0.2656	0.797911	2.008571	2.394195	2.08052	1.756991	3.998806	2.447817	232.3425
0.111762	0.450642	0.389337	0.641523	0.595297	1.088651	2.447457	2.220835	3.202955	3.007659	2.393511	234.736
0.389022	0.341063	0.492645	0.802348	0.80258	2.219673	2.042371	2.604888	4.02647	4.027928	2.984266	237.7203
0.204575	0.103531	0.645733	0.499079	0.395474	1.512872	1.045426	3.221344	2.629273	2.243458	2.130474	239.8508
0.842562	0.409461	0.297681	0.665977	0.708492	4.299676	2.295036	1.879807	3.311411	3.510967	3.059379	242.9101
0.174126	0.376722	0.762366	0.175909	0.900089	1.383159	2.174311	3.790811	1.390951	4.799454	2.707737	245.6179
0.4726	0.437003	0.822258	0.099824	0.178302	2.52942	2.396834	4.156228	1.025498	1.401364	2.301869	247.9198
0.459174	0.30398	0.055246	0.365261	0.441918	2.479228	1.903618	0.753863	2.131998	2.415055	1.936753	249.8565
0.632511	0.462599	0.80675	0.78342	0.72293	3.163955	2.492009	4.054344	3.911259	3.582577	3.440829	253.2973
0.607179	0.87252	0.577212	0.934602	0.511025	3.056797	4.538501	2.934082	5.222322	2.67478	3.685296	256.9826
0.22662	0.678441	0.494093	0.474229	0.482517	1.603073	3.368344	2.610369	2.535527	2.56667	2.536797	259.5194
0.114439	0.267281	0.497887	0.02344	0.02489	1.102422	1.763499	2.624749	0.487024	0.50205	1.295949	260.8154
0.997366	0.870752	0.057519	0.104392	0.825572	7.706598	4.523302	0.769673	1.050009	4.178806	3.645678	27.83186
0.312162	0.162079	0.826692	0.825268	0.357745	1.934431	1.329782	4.186508	4.176722	2.104209	2.74633	30.57819
0.639841	0.579459	0.310738	0.799061	0.269727	3.195639	2.94315	1.929077	4.005936	1.772955	2.769351	33.34755
0.226971	0.588153	0.678421	0.878056	0.480752	1.604488	2.978428	3.368254	4.587149	2.560026	3.019669	36.36721
0.991931	0.617915	0.496611	0.4424	0.760028	6.942421	3.101794	2.61991	2.416843	3.777874	3.771769	40.13898
0.504207	0.497818	0.992256	0.454943	0.418513	2.648766	2.624484	6.971997	2.463464	2.328449	3.407432	43.54641
0.361082	0.960061	0.560078	0.89355	0.858143	2.11655	5.674851	2.865585	4.732944	4.4192	3.961826	47.50824
0.618456	0.112533	0.002072	0.043901	0.828911	3.104077	1.092629	0.144008	0.670025	4.201868	1.842521	49.35076
0.539092	0.513114	0.643304	0.461323	0.945145	2.783084	2.68277	3.210717	2.487244	5.388015	3.310366	52.66113
0.15766	0.812563	0.014391	0.857294	0.628503	1.309853	4.091834	0.380736	4.412444	3.146766	2.668327	55.32946
0.089881	0.592489	0.167419	0.190034	0.840141	0.970463	2.996143	1.353605	1.45177	4.281894	2.210775	57.54023
0.045086	0.784834	0.020759	0.030218	0.369248	0.679224	3.919622	0.458013	0.553933	2.146725	1.551503	59.09173
0.226718	0.670126	0.574915	0.778096	0.206179	1.60347	3.33023	2.924835	3.88009	1.519532	2.651632	61.74337
0.234904	0.283537	0.946452	0.789951	0.189433	1.636321	1.826003	5.410337	3.950209	1.449211	2.854416	64.59778
0.067033	0.069649	0.159669	0.816753	0.175956	0.832982	0.849664	1.318936	4.11937	1.391156	1.702422	66.3002
0.387598	0.624917	0.149575	0.908453	0.644392	2.214422	3.131468	1.272868	4.889688	3.215473	2.944784	69.24499

0.693466	0.444261	0.220666	0.743877	0.532436	3.438644	2.42375	1.578974	3.690662	2.757208	2.777848	72.02283
0.868248	0.692413	0.868594	0.569176	0.109653	4.502038	3.433654	4.504958	2.901819	1.077704	3.284035	75.30687
0.239514	0.043669	0.735508	0.532837	0.148688	1.654683	0.668216	3.646838	2.758763	1.268765	1.999453	77.30632
0.760774	0.594362	0.758587	0.457238	0.711159	3.781995	3.00382	3.769939	2.472012	3.524031	3.310359	80.61668
0.381752	0.782937	0.360872	0.478459	0.615222	2.192866	3.908413	2.115777	2.551406	3.090453	2.771783	83.38846
0.042588	0.349615	0.129594	0.065386	0.953029	0.659704	2.074104	1.178115	0.822322	5.530124	2.052874	85.44134
0.721949	0.703315	0.450675	0.091028	0.27676	3.577642	3.485804	2.447579	0.97694	1.800039	2.457601	87.89894
0.91814	0.524971	0.25656	0.534386	0.13822	5.002738	2.728334	1.721822	2.764776	1.219654	2.687465	90.5864
0.29909	0.458878	0.182834	0.146905	0.923206	1.885142	2.478125	1.42096	1.260493	5.066196	2.422183	93.00859
0.442683	0.411014	0.688133	0.398941	0.65649	2.417894	2.300766	3.413472	2.25624	3.268854	2.731445	95.74003
0.893405	0.408221	0.364987	0.706901	0.882937	4.731511	2.290464	2.130986	3.503207	4.631462	3.457526	99.19756
0.52187	0.9746	0.917668	0.791722	0.957215	2.716382	6.060526	4.996996	3.960909	5.613893	4.669741	103.8673
0.560007	0.886869	0.870727	0.23386	0.748548	2.865305	4.668203	4.523082	1.632148	3.71551	3.48085	107.3481
0.112261	0.905135	0.292407	0.786138	0.224599	1.091227	4.85314	1.859802	3.927369	1.594913	2.66529	110.0134
0.925141	0.265613	0.551236	0.959546	0.682931	5.091318	1.757039	2.830652	5.663561	3.389155	3.746345	113.7598
0.877053	0.559834	0.226323	0.393631	0.283629	4.57821	2.864618	1.601878	2.236662	1.826356	2.621545	116.3813
0.809859	0.948636	0.996824	0.514056	0.669832	4.074294	5.44868	7.584275	2.686376	3.328892	4.624503	121.0058
0.401266	0.918872	0.48654	0.947274	0.622171	2.264815	5.011713	2.581828	5.424611	3.119796	3.680553	124.6864
0.145343	0.252734	0.480365	0.910569	0.925586	1.253214	1.706849	2.55857	4.913536	5.097173	3.105869	127.7923
0.51097	0.972339	0.129542	0.893388	0.41399	2.674569	5.989774	1.177863	4.731339	2.31175	3.377059	131.1693
0.660173	0.407619	0.001886	0.26989	0.7929	3.285298	2.288242	0.137388	1.773586	3.968063	2.290516	133.4598
0.205896	0.828186	0.157613	0.743535	0.601043	1.518357	4.196836	1.30964	3.688853	3.031341	2.749005	136.2088
0.331402	0.386504	0.032681	0.983167	0.562905	2.00642	2.210389	0.576431	6.390933	2.87681	2.812197	139.021
0.457046	0.179586	0.321599	0.582447	0.059432	2.471296	1.406934	1.969813	2.955238	0.782757	1.917208	140.9382
0.383342	0.289396	0.569169	0.03471	0.776164	2.198728	1.848351	2.901793	0.594366	3.868906	2.282429	143.2207
0.124225	0.910454	0.024733	0.772928	0.342578	1.151719	4.912233	0.500442	3.850311	2.047998	2.49254	145.7132
0.669742	0.786803	0.694441	0.706499	0.979952	3.328485	3.931332	3.443272	3.501249	6.25269	4.091406	149.8046
0.550382	0.881991	0.995291	0.472601	0.565711	2.827291	4.62276	7.319992	2.529424	2.887983	4.03749	153.8421
0.400579	0.674753	0.162316	0.503052	0.7886	2.26228	3.351375	1.330842	2.644371	3.942086	2.706191	156.5483
0.936353	0.277682	0.358262	0.230677	0.43909	5.248246	1.803579	2.106122	1.619396	2.404568	2.636382	159.1847
0.315556	0.279618	0.773527	0.50577	0.237518	1.947172	1.811004	3.853736	2.654721	1.646744	2.382675	161.5674
0.673332	0.475239	0.26856	0.80238	0.326579	3.344862	2.539314	1.768445	4.026675	1.988429	2.733545	164.3009
0.566819	0.029201	0.803045	0.149194	0.13943	2.892402	0.544392	4.030857	1.271108	1.225397	1.992831	166.2937
0.787743	0.652796	0.579165	0.567044	0.828219	3.936951	3.25245	2.941961	2.893301	4.197064	3.444345	169.7381
0.735807	0.686101	0.78536	0.445988	0.027486	3.648388	3.403947	3.922748	2.430164	0.527927	2.786635	172.5247
0.768046	0.814432	0.31259	0.217817	0.697967	3.822585	4.104063	1.936037	1.567373	3.460086	2.978029	175.5027
0.112654	0.740557	0.388219	0.451569	0.749176	1.093256	3.673168	2.21671	2.450907	3.718876	2.630583	178.1333
0.810843	0.84033	0.328396	0.183545	0.224922	4.080661	4.28328	1.995209	1.424019	1.59622	2.675878	180.8092
0.031541	0.645001	0.03614	0.045369	0.50277	0.566121	3.218138	0.606702	0.681395	2.643298	1.543131	182.3523
0.974439	0.448752	0.58348	0.892001	0.382132	6.055328	2.440433	2.959427	4.717662	2.194267	3.673423	186.0258
0.98722	0.408294	0.503055	0.297204	0.980399	6.602933	2.290731	2.644383	1.878001	6.270696	3.937349	189.9631

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