Abstract
One of the macroeconomic objective is to have price stability. The quantity theory of money (QTM) explains the nexus between money supply and price level in the economy thus, QTM provides the theoretical framework for monetary policy to bring price stability. Therefore, this study investigates the validity of QTM in Pakistan in time series study. This study applied different econometric techniques, unit root tests, cointegration test, and fully modified ordinary least square (FMOLS) cointegration regression. The variables money supply, price level, interest rate and output are cointegrated in the long run. The results of the FMOLS regression indicate positive and significant effect of money supply on price level henceforth; this study validate the QTM in Pakistan. The study suggests that monetary policy can effectively control the price level in the economy to maintain economic stability and sustain economic growth.

Keywords: money supply, price level, interest rate, output

1. Introduction
The classical QTM explains the relationship between the supply of money and the price level. The beginnings of the QTM can be traced back to Bodin (1568), who first proposed his monetary theory of inflation brought on by fresh monetary metal inflows from South America. Numerous economists have since contributed to the theory, including Locke (1692), Hume (1752), Brunner and Meltzer (1963), Friedman (1968), and Mises (1912). The QTM is based on Fisher's (1911) equation of exchange, which asserts that the nominal GDP is equal to money supply multiplied by the velocity of money. Both the velocity of money and real income are anticipated to be fixed in the short period under the most basic form of the equation. Changes in the money supply over time affects the level of prices proportionately (Lee & Huruta, 2021; Roussel et al., 2021). According to the QTM, there is a direct association between variations in the supply of money and changes in the buying power of money. It contends that changes in the velocity at which money circulates in the economy play a significant part in determining variation in the price level. As a result of the negative consequences that high price level have on an economy, the top central banks in the world concurs that price stability should be the primary objective of monetary policy (Bhowmik, 2020). Price stability is among the primary goals of the twenty-first-century economic system (Muhammad et al., 2013; Inim et al., 2020). A low inflation rate is highly favored because it may motivate businesses to produce more output, raise the standard of living in society, and enable greater savings (Mala et al., 2021).

For a well-functioning economy, price stability is very important because it increases the transparency of the pricing process, which boosts economic activity and employment. It promotes financial stability, increases the value of cash held, reduces uncertainty about general price rises, lowers the risk associated with inflation, and aids in the avoidance of unnecessary hedging: it increases the value of cash held, stops the arbitrary distribution of income and wealth; and it can achieve a wide range of other economic objectives (Regret & Simbisai, 2020). In recent years, Pakistan has experienced relatively high inflation rates. Several variables, including rising food and energy costs, currency devaluation, and an expansionary fiscal policy, have contributed to inflation. In response to inflationary pressures, the State Bank of Pakistan (SBP) raised interest rates and tightened monetary policy. It is critical to remember that the field of monetary policy is complex and ever-evolving and the specific policies and tools used by the SBP to control the money supply and prices in Pakistan can vary over time. A crucial economic indicator, inflation offers important information about general price patterns and the overall cost of living. Price stability is crucial because it promotes economic growth and helps vulnerable people, especially low-income earners. In emerging countries like Pakistan, a stable inflation environment is necessary to promote attractive savings and investments, which foster wide-ranging and sustained growth (SBP, 2022).

The COVID-19 epidemic, which began in 2020, struck the world and sparked a disaster. To avoid an economic recession, governments and central banks around the world adopted previously unheard-of actions, such as adopting accommodating fiscal and monetary policies. Despite these efforts, the necessary restraints had a negative influence on social and economic progress, which led to a recession in 2020. The crisis between Russia and Ukraine at the beginning of 2022 increased commodities prices globally, which increased domestic inflation rates and inflation. These risks, together with supply limitations and escalating unpredictability, caused the prediction for global growth to be revised downward. The prediction was lowered by 0.8 and 0.2 percentage points in 2022 and 2023, respectively. The estimate for the developed and developing economies was also revised downward by 0.61 and 1.01 percentage points, respectively, while the European economies witnessed a 1.1-point decline to 2.8 percent. All categories' medium-term outlooks were downgraded, except for commodity exporters who stand to gain from rising energy and food costs (ESP, 2022).

Despite having a target inflation rate of 8.0 percent for the current fiscal year, Pakistan's domestic prices have increased due to unusually high rises in the pricing of global commodities, particularly crude and edible oil. Pakistan has suffered greatly since it is a net importer of these necessary commodities. Inflation has been in the double digits for six consecutive months, with the
Consumer Price Index (CPI) rising from 12.7 percent in April 2021 to 13.4 percent in April 2022. Regarding food inflation in the same month, urban areas had an increase of 15.6 percent while rural areas saw an increase of 17.7 percent. From April to July of 2022, the CPI inflation rate was 11.01 percent on average, up from 8.61 percent at the same duration in 2021 (SBP, 2022). Stability of Prices is the key objective of the monetary policy. Stability of Prices boosts economic activity and increases employment opportunities; it promotes financial stability in the economy, decreases uncertainty about general price levels, and encourages a high standard of living. This investigation will examine the validity of QTM and determine whether it is valid or not in the context of Pakistan; likewise, the outcome of the study will also determine whether the stability of prices was influenced by monetary policy in Pakistan or not.

2. Literature review
The QTM, which claims that the amount of money is the sole factor that influences the level of prices, was initially put forth by Bodin in 1566. In addition, Hume (1752) argued that deflation results from a decline in the money supply whereas inflation results from an increase in money supply. When a country uses the gold standard, the price-specie-flow technique is used, where an increase in trade raises the money supply and a deficit in trade lowers it. By adding paper money and fractional reserve banking to the theory's framework, Thornton (1802) pushed contemporary politicians to defend institutional practices. According to the theory, money is neutral in influencing real variables and money is the main factor that determines the price level. According to Fisher (1911) and Pigou (1917), the QTM takes the identity equation MV=PT, the total money supply is multiplied by its velocity, while the total amount of money spent is equal to the total value of all transactions. However, in practice, this relationship may be asymmetric or have no impact in some cases, while the association between money supply and price level has been the subject of numerous research studies.

Matres and Le (2021) used a panel of 217 countries to study the relationship between money supply inflation, exchange rates, economic growth, and real interest rates between 1960 and 2020. The findings validate QTM. Furthermore, inflation and interest rate, as well as interest rate and the money supply, were found to be negatively correlated. To determine nexus between money supply and price level, Pinter (2021) conducted a study and found that there is a direct correlation between money supply and price level. Furthermore, it is suggested that it is the main factor for demand for money, and high inflation are not anticipated to result from the rapid money growth rates that have been seen since the pandemic breakout. Stewart (2021) examined QTM by employing limits tests for the U.S. economy. The results, which reveal a one-to-one relationship between prices and money or income when \( M_2 \) is the monetary aggregate, are typically supported by 100-year data from the United States. Using interest rates experienced by the United States over the previous century and also taking into account for structural change, this study concluded that the overall cost of inflation varies from 0.362 to 1.326 percent of the gross domestic product. Bojanic (2021) employed a time-varying multivariate as well as univariate Markov switching model to study the Bolivian inflation process. The results of this investigation were consistent with what the QTM had predicted. The study also found that GDP growth accounted for the majority of variations in inflation, particularly during periods of high inflation. Chang (2021) evaluated the validity of QTM and put lagged \( M_2 \) money supply capacity to predict US real GDP. The study concluded that money supply granger causates real GDP but not the real GDP causes money supply.

Hassan and Teleb (2022) investigated the factors of money supply in Egypt. The research shows that despite the exchange rate and net domestic reserves having little effect on the money supply, real GDP, the rate of discount, the fiscal deficit, and net foreign assets have all significant long-term effects. The quantity equation is criticized by Avdou and Unger (2022) because it is a long-term theory and cannot make precise claims about the dynamics or causality between money expansion and inflation. Alternatively, they recommend estimating inflation systemically by taking into consideration varying levels of GDP, supply of money, and velocity. They found that productivity is the primary cause of inflation, even though money supply play a big role as well.

Roussel, Ali, and Audi (2021) have looked into the factors that influenced Pakistani money demand between 1980 and 2019. The authors used a variety of independent factors, such as household and government consumption, interest rates, the consumer price index, population growth, and remittances, along with broad money as the dependent variable. The authors employed the Phillip Perron (PP) and Augmented Dickey-Fuller (ADF) unit root tests to investigate the stationarity of the variables. They looked for causal linkages between the variables using the Granger Causality test and the ARDL technique to evaluate for cointegration between the variables. The variable's asymmetrical order of integration supports the study’s results that money demand in Pakistan is significantly influenced by socioeconomic characteristics. To facilitate daily transactions for enterprises, firms, and people, money is a crucial asset. The factors that influence money demand are the subject of numerous hypotheses from diverse economic schools of thought. For example, neo-classical economists claim that nominal income GDP affects the demand for money, anticipated rates of return on investments, bonds, equities, and durable commodities. According to the study, socioeconomic considerations are very important in determining Pakistan's desire for money. In a panel study, Bilgrami and Maryam (2022) empirically investigate inflation in Bangladesh, India, Sri Lanka, and Pakistan, with the use of the Robust System GMM technique. They concluded that current account balances, exchange rate, and interest rate lead to inflation in these countries.

3. Research Methodology
This time series study is to test the application of QTM in the context of Pakistan from the period 1964 to 2021. This sample period is selected upon the availability and quality of the data for a longer period, a larger sample size gives more accurate results as compared to a short period and QTM can be tested for a longer period. Data were gathered from Federal Bureau of Statistics, the State Bank of Pakistan, International Monetary Fund, and World Bank online databases.
A key idea in monetary economics is the connection between the money supply and the rate of inflation. The QTM which holds that the amount of money in circulation and its velocity are closely correlated with inflation, is a common theory used to explain the reason for this association. The equation of QTM is as:

\[ M \cdot V = P \cdot Y \]  

(1)

Where M, V, Y and P denote money stock, the velocity of money in circulation, output level and price level, respectively. Taking the natural logarithm of equation (1), we get:

\[ \ln M_t + \ln V_t = \ln P_t + \ln Y_t \]  

(2)

The simplest method to convert this identity into a form of testable hypothesis is by considering the constant velocity of money in circulation. In this study, the velocity of money in circulation is transformed into the equation of the rates of discount rate, as proposed by Emerson (2006).

\[ \ln V_t = \alpha_0 + \alpha_1 \ln DR_t + e_t \]  

(3)

Where, \( \alpha_0 \) and \( \alpha_1 \) are coefficients, DR and \( e_t \) denote discount rate and error term, respectively. Equations (2) and (3) are combined and rearranged to yield:

\[ \ln P_t = \ln M_t + \alpha_0 + \alpha_1 \ln DR_t - \ln Y_t + e_t \]  

(4)

Equation (4) can be simplified as:

\[ \ln P_t = \beta_0 + \beta_1 \ln M_t + \beta_2 \ln DR_t + \beta_3 \ln Y_t + e_t \]  

(5)

Where \( \beta_0 \) represents the intercept, \( \beta_1 \), \( \beta_2 \), and \( \beta_3 \) represent the money supply, discount rate, and output coefficients, respectively, and \( e_t \) the error term of the regression model.

When analyzing time series data, unit root testing is a crucial tool for determining if the series has unit root issues or not. Non-stationary time series have statistical characteristics that change over time, while stationary time series data exhibit constant statistical qualities across The most popular method for testing a unit root in time series data is the ADF test developed by (Dickey & Fuller, 1979). The other widely used test is the Phillips-Perron (PP) test. A statistical technique called cointegration analysis is employed for investigating the long-term relationship between two or more time series data that are not stable. Cointegration is based on the theory that even if two variables could separately be non-stationary, they may still share a common long-run trend or equilibrium relationship. A common Johansen cointegration test which is suggested by Johansen is employed for checking the long-run cointegration or equilibrium relation among the time series variables. In this case, the Johansen cointegration test is employed to find the equilibrium association among the price level, money supply, interest rate, and output for verifying the application of QTM in the context of Pakistan. Once cointegration is determined then next step is to find long run results based on cointegration regression. This study uses fully modified ordinary least squares (FMOLS). This cointegration regression technique gives consistent and efficient estimates (Phillips & Hansen, 1990). This technique has various advantages over other techniques as it taking into account to endogeneity problem that may arise because of cointegration process.

4. Results and Discussion

Table 1 reveals the results unit root tests (ADF and PP). Unit root time series has variations in mean and variance over time in contrast to stationary time series that has constant mean and variation over time. The table shows the outcomes of the Phillips-Perron (PP) and the (ADF) test for the four variables price level, velocity of money, money supply, and output (GDP). The results are provided for both the variable's level and its initial difference. The results could be useful in selecting the best statistical models and testing procedures for time series data. The findings of both experiments reveal that while all four variables have unit roots at the level, they become stationary by taking the first difference of the series.

<table>
<thead>
<tr>
<th>Variables</th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>At Level</td>
<td>At 1st Difference</td>
</tr>
<tr>
<td>( \ln P )</td>
<td>-0.501 (0.882)</td>
<td>-3.563 (0.009)</td>
</tr>
<tr>
<td>( \ln Int )</td>
<td>-0.203 (0.608)</td>
<td>-6.057 (0.000)</td>
</tr>
<tr>
<td>( \ln M )</td>
<td>-0.477 (0.887)</td>
<td>-5.983 (0.000)</td>
</tr>
<tr>
<td>( \ln Y )</td>
<td>-1.199 (0.901)</td>
<td>-6.975 (0.000)</td>
</tr>
</tbody>
</table>

| Table 2: The results of cointegration |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Rank \( r \) | Trace Statistics | 0.05 Critical Value | Prob.** | Maximum Eigenvalue | 0.05 Critical Value | Prob.** |
| \( r_0 = 0 \) | 59.148 | 47.856* | 0.003 | 28.820 | 27.584* | 0.034 |
| \( r_0 \leq 1 \) | 30.328 | 29.797* | 0.043 | 20.082 | 21.131 | 0.069 |
| \( r_0 \leq 2 \) | 10.246 | 15.494 | 0.262 | 6.754 | 14.264 | 0.518 |
| \( r_0 \leq 3 \) | 3.491 | 3.841 | 0.061 | 3.491 | 3.841 | 0.061 |

Table 2 shows the findings of a cointegration test between two or more variables. The cointegration test is used to test whether a linear combination of these variables is stationary. This indicates that no long-term trends have an impact on the combination.
There are two often-used techniques for determining how many cointegrating equations there are in a system of variables: the trace test and the maximum Eigenvalue test. The table compares the test statistics with their critical values at a significance level of 0.05 and shows the test results for various hypotheses (i.e., different numbers of cointegrating equations). In this case, the Maximum Eigenvalue test reveals just one cointegrating equation whereas the Trace test finds two at the level of 0.05 of significance.

The results of the FMOLS test are shown in Table 3, where the level of prices is the dependent variable and there are three independent variables: supply of money, velocity of money (interest rate), and output (GDP). The coefficient for the money supply is 0.693, the t-statistic is 53.718, and the probability is (P<1%). The results show that the price level increases by 0.69% for every one percent rise in the money supply, and that the relationship is highly significant at the one percent level of significance. It demonstrates that if the amount of money available increases, prices will also rise or vice versa. The results are similar with the findings of the studies of Farooq, Hassan, and Shahid (2015), Yuliadi (2020) and Bhowmik (2020). The findings reveal that interest rate has a coefficient 0.19 with a probability value (P < 1%). The results indicate that one change in the interest rate will change price level by 0.19 percent. The findings are similar with the study conducted by (Yuliadi, 2020; Wang, 2020; Prihatin, et al., 2019; Gatawa, Abdulgafar & Olarinde, 2017; Pinter, 2021; Mohamed, 2020; Akeerebari, 2022).

Output (GDP) has a coefficient of 0.397 and, a t-statistic value is 27.664 with a probability value (P < 1%). It demonstrates that it is highly significant at the 1% level of significance and that there is a 0.39% rise or fall in the price level in the same direction for every 1% change in the GDP (output). It shows that if there is a rise in the Output (GDP), so there is an increase in the price level, or vice versa. The conclusion supports the findings of the research of (Hassan and Teleb, 2022). The coefficient of determination (R-squared) statistical measures the variation in dependent variable due to variation in independent variables. The R-squared value of 0.79 indicates that the independent variables in this situation, the money supply, interest rate, and real production (GDP) explain 79% of the variance in the price level.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnM</td>
<td>0.693</td>
<td>0.013</td>
<td>53.718</td>
<td>P &lt; 1%</td>
</tr>
<tr>
<td>lnInt</td>
<td>0.189</td>
<td>0.066</td>
<td>2.832</td>
<td>P &lt; 1%</td>
</tr>
<tr>
<td>lnY</td>
<td>0.394</td>
<td>0.014</td>
<td>27.664</td>
<td>P &lt; 1%</td>
</tr>
<tr>
<td>Constant</td>
<td>4.023</td>
<td>0.957</td>
<td>4.206</td>
<td>P &lt; 1%</td>
</tr>
</tbody>
</table>

R-squared 0.79 Adjusted R-squared 0.78

After running a cointegration regression it becomes necessary to check whether the results obtained from the model are consistent and free from problems and satisfy the assumption of a classical regression model. Therefore, serial correlation test, multicollinearity, and heteroscedasticity tests are employed. Diagnostic tests results are given in Table 4. For checking the presence of an outlier the normality test is applied and the results confirm the normality of data, for checking the multicollinearity among the independent variables the multicollinearity test is employed and confirms the results by Variance Inflation Factors (VIFs), The absence of heteroscedasticity and serial correlation for these variables is confirmed by the findings of the Breusch-Pagan Godfrey Heteroscedasticity Test, and the Breusch-Godfrey Serial Correlation LM Test.

<table>
<thead>
<tr>
<th>Test</th>
<th>F-stat.</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial Correlation LM Test</td>
<td>1.978</td>
<td>0.174</td>
</tr>
<tr>
<td>Heteroscedasticity Test</td>
<td>0.871</td>
<td>0.582</td>
</tr>
<tr>
<td>Multicollinearity Test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>Coefficient</td>
<td>Uncentered VIF</td>
</tr>
<tr>
<td>lnInt</td>
<td>0.012</td>
<td>1.451</td>
</tr>
<tr>
<td>lnM</td>
<td>0.024</td>
<td>1.873</td>
</tr>
<tr>
<td>lnY</td>
<td>0.013</td>
<td>2.024</td>
</tr>
</tbody>
</table>

5. Conclusions
The primary aim of this study is to test the application of QTM in the context of Pakistan from the period 1964 to 2021. The impact of money supply, interest rate, and output (GDP) on the level of prices are tested by employing different econometric techniques. The results of the ADF and Phillips-Perron (PP) test show that the supply of money, price level, interest rate, and output are all stationary at the first difference. The long run cointegration is tested through Johansen Cointegration technique and results of this technique confirmed the long run cointegration among the variables. The results of the FMOLS test conclude the positive and long-run impact of the supply of money, interest rate, and output (GDP) on the price level in the context of Pakistan. After the utilization of different econometric techniques for the testing of the application of QTM in Pakistan, and confirmation of its application in the context of Pakistan, the recommendations are as follows: To control the rate of inflation in Pakistan, policymakers are required to devise policies that control the excessive supply of money and take measures that control the price level. It suggests that policymakers pay special attention to the factors that impact the velocity of circulation of money which enhances the efficiency of financial transactions and encourage consumption spending. It is suggested to devise such monetary and
fiscal policies that increase productivity which causes economic growth in the economy by lowering the price level. It is recommended to devise fiscal, monetary, and other economic policies, and coordination among these policies is essential that reduce the inflationary pressure in the economy and encourage economic growth. For controlling the money supply there is a need to adjust monetary policy instruments for the economic stability of the economy. It is recommended that key economic indicators like price level, supply of money, and other relevant variables must be monitored regularly.

References
Locke J. (1692), *Consequences of the Lowering of Interest and Raising the Value of Money*. Online available at: https://fqod.lib.umich.edu/e/ebo/848895.0001.001?view=toc.


